

# Percy Ludgate's Logarithmic Indexes

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## A. Ludgate's logarithmic indexes

Percy Ludgate invented his own logarithmic indexes for multiplication [1][2][3], an entirely new result in 1909. These are now called Irish Logarithms, a discrete analogy to real logarithms where  $\log(j*k) = \log(j) + \log(k)$ . For two operands  $Z_J$  and  $Z_K$  the index numbers ensure  $Z_Y = Z_{J*K} = Z_J + Z_K$ , for example  $Z_3 = 7$ ,  $Z_5 = 23$ ,  $Z_{15} = 30$ . The largest  $Z_{J*K}$  is  $Z_{7*7} = 66$ . C.V.Boys said: "Ludgate ... uses for each of the prime numbers below ten in a logarithmic system with a different incommensurable base, which as a fact never appears" [2], i.e. each  $Z_X = \log_{N_X}(X)$  has a different (invisible) base  $N_X$ . Ludgate almost certainly did not derive his indexes from number theory, either finding them empirically or by a systematic "method", such as that in Figure 1.

Ludgate's proposed logarithmic indexes are amenable to construction by a systematic method.

For an example of how these are used in calculations, see Andries de Man's educational emulator [5], and his citations of similar indexes in the literature [6] cited below:

- (1) Jacobi/Zech indexes (1846/1849 [7]), which can be derived from number theory. For example  $Z_1 = 0$ ,  $Z_2 = 1$ ,  $Z_3 = 18$ ,  $Z_5 = 44$ ,  $Z_7 = 7$ . An alternative is  $Z_1 = 0$ ,  $Z_2 = 1$ ,  $Z_3 = 8$ ,  $Z_5 = 44$ ,  $Z_7 = 27$ . In both cases the largest  $Z_{J*K}$  is  $Z_{5*5} = 88$ .
- (2) Remak indexes (Von K. Hoecken 1913 [8]),  $Z_1 = 0$ ,  $Z_2 = 1$ ,  $Z_3 = 13$ ,  $Z_5 = 21$ ,  $Z_7 = 30$ , where the largest  $Z_{J*K}$  is  $Z_{7*7} = 60$ .
- (3) Korn indexes (Von K. Hoecken 1913 [8]),  $Z_1 = 0$ ,  $Z_2 = 8$ ,  $Z_3 = 13$ ,  $Z_5 = 1$ ,  $Z_7 = 30$ , where the largest  $Z_{J*K}$  is  $Z_{7*7} = 60$ .

Ludgate said: "The index numbers (which I believe to be the smallest whole numbers that will give the required results)". His focus on small numbers makes it possible Ludgate knew of Jacobi/Zech indexes, which were in use in astronomy. Remak and Korn indexes give smaller numbers, but were reported after Ludgate had published. Small numbers are important to minimize the length of any mechanism that is used to convert to or from the logarithmic indexes. McQuillan [11][12] has shown that better indexes can be readily be computed using modern machinery.

Ludgate proposed logarithmic indexes with the smallest  $Z_{J*K}$  then known.

As happens surprisingly often with novel inventions, in the same year as Ludgate's 1909 paper a sliderule with Jacobi indexes was designed by Prof.Schumacher of Germany [9] and later manufactured as the Faber Model 366 [10].

Ludgate's indexes can be derived by an algorithm. He likely did this, but thought of it as a "method". The method can be expressed as a simple algorithm, see below, which has proven Irish Logarithms exist over the range 0-99, perhaps further.

Ludgate's proposed logarithmic indexes are amenable to algorithmic construction.

Ludgate's logarithmic indexes have been shown to exist for 0-99.

## B. Algorithm to derive Irish Logarithms

Ludgate's indexes can be derived by algorithm. He likely did this, but thought of it as a "method". The method of Figure 1 can be expressed as a simple algorithm, for example in Python as in Figure 2. Executing this algorithm yields identical indexes to those given in Tables 1, 2 and 3 of Ludgate's 1909 paper, see Figure 3.

Irish Logarithms might exist for the range up to any  $N$ , not just for 0-9 as in Ludgate's paper. Certainly they have been shown to exist for 0-99 by extending the rather inefficient algorithm of Figure 2 with a basic Sieve of Eratosthenes to generate the seed prime numbers. Beyond that a more efficient algorithm would be necessary, or ideally a number-theoretic proof.



```
#!/usr/bin/env python

import sys

# initialise variables
Z=[-1]*200 # table of complex indexes
PP=[-1]*200 # table of partial products
i = 0;
for p in (1,2,3,5,7,0):
    if Z[p]==-1: # prime not indexed yet
        free=False
        while free==False and i<=100:
            free=True
            for j in (1,2,3,4,5,6,7,8,9):
                if free==True:
                    if Z[j]<>-1: # for existing indexes
                        for k in range (1,100):
                            if Z[k]==(i+Z[j]): # check if complex index exists
                                free=False
                                i=i+1
            if free==True: # OK, found a desired free set of indexes
                Z[p]=i # create new simple index
                PP[i]=p # create new partial product
                i=i+1
            for j in (1,2,3,4,5,6,7,8,9,0):
                if Z[j]<>-1: # multiplicand simple index exists
                    for k in (1,2,3,4,5,6,7,8,9,0):
                        if Z[k]<>-1: # multiplier simple index exists
                            if PP[Z[j]+Z[k]]==-1: # product not indexed yet
                                if Z[j*k]==-1:
                                    Z[j*k]=Z[j]+Z[k] # create new complex index
                                    PP[Z[j]+Z[k]]=j*k # create new partial product
```

**Figure 2** Python algorithm to derive Ludgate's simple index numbers  
Image reproduced courtesy The John Gabriel Byrne Computer Science Collection [4]

```
final Ludgate Simple Index for each Unit (Table 1):
0: 50  1:  0  2:  1  3:  7  4:  2  5: 23  6:  8  7: 33  8:  3  9: 14

final Ludgate Complex Index for each Partial Product (Table 2):
1:  0  2:  1  3:  7  4:  2  5: 23  6:  8  7: 33  8:  3  9: 14 10: 24 12:  9 14: 34
15: 30 16:  4 18: 15 20: 25 21: 40 24: 10 25: 46 27: 21 28: 35 30: 31 32:  5 35: 56
36: 16 40: 26 42: 41 45: 37 48: 11 49: 66 54: 22 56: 36 63: 47 64:  6 72: 17 81: 28

final Ludgate Partial Product for each Complex Index (Table 3):
0:  1  1:  2  2:  4  3:  8  4: 16  5: 32  6: 64  7:  3  8:  6  9: 12
10: 24 11: 48 12:  13:  14:  9 15: 18 16: 36 17: 72 18:  19:
20:  21: 27 22: 54 23:  5 24: 10 25: 20 26: 40 27:  28: 81 29:
30: 15 31: 30 32:  33:  7 34: 14 35: 28 36: 56 37: 45 38:  39:
40: 21 41: 42 42:  43:  44:  45:  46: 25 47: 63 48:  49:
50:  0 51:  0 52:  0 53:  0 54:  55:  56: 35 57:  0 58:  0 59:
60:  61:  62:  63:  64:  0 65:  66: 49 67:  68:  69:
70:  71:  72:  73:  0 74:  75:  76:  77:  78:  79:
80:  81:  82:  83:  0 84:  85:  86:  87:  88:  89:
90:  91:  92:  93:  94:  95:  96:  97:  98:  99:
100:  0
```

**Figure 3** Results from running the algorithm of Figure 2  
Image reproduced courtesy The John Gabriel Byrne Computer Science Collection [4]

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