Speculations on Percy Ludgate's Difference Engine

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Abstract In Percy Ludgate's 1914 paper for the Napier Tercentenary Exhibition he briefly mentioned that he had designed a simplified difference engine. This paper speculates on how that might have worked. **This paper is a work in progress**.

***Index Terms*—Percy Ludgate, Analytical Machine, Difference Engine**

I. INTRODUCTION

PERCY Edwin Ludgate (1883-1922) is notable as the second person to publish a design for an Analytical Machine [1][2], the first after Charles Babbage's "Analytical Engine" [4]. An analytical machine is equivalent to a general-purpose computer, and in theory can be programmed to solve any solvable problem. It is "Turing complete", a term invented to reflect Alan Turing's contribution to the theory of computing. There were only two mechanical designs before the electronic computer era: in 1843 Babbage's "analytical engine", then in 1909 Ludgate's very different "analytical machine".

Babbage only started work on his Analytical Engine in c.1834 when his intensive efforts to design and make a difference engine faltered. This machine was intended to evaluate polynomials using Newton's method of divided differences in order to produce mathematical and nautical tables. In 1914 Ludgate published a paper for the Napier Tercentenary Exhibition [3] (hereafter called "Ludgate 1914") in which he briefly mentioned that he had designed a simplified difference engine. This paper speculates on how that might have worked.

II. DIFFERENCE ENGINES

II.1. Babbage's Difference Engine

Charles Babbage is shown in Figure 2(a), drawn late in his life. He is historically important as the first person to design a difference engine and then also the first to design an analytical engine [22]. From Ludgate 1914:

> The first great automatic calculating machine was invented by Charles Babbage. He called it a "difference-engine," and commenced to construct it about the year 1822. The work was continued during the following twenty years, the Government contributing about £17,000 to defray its cost, and Babbage himself a further sum of about £6ooo. At the end of that time the construction of the engine, though nearly finished, was unfortunately abandoned owing to some misunderstanding with the Government. A portion of this engine is exhibited in South Kensington Museum, along with other examples of Babbage's work. If the engine had been finished, it would have contained seven columns of wheels, with twenty wheels in each column (for computing with six orders of differences), and also, a contrivance for stereotyping the tables calculated by it. A machine of this kind will calculate a sequence of tabular numbers automatically when its figure-wheels are first set to correct initial values.

Babbage actually designed two difference engines. His difference engine no.1 operated on 20-digit numbers and sixth-order differences. His difference engine no.2 operated on 31-digit numbers and seventh-order differences.

To support (N-1)-order differences, his difference engines consisted of N columns, each storing one decimal number, where each column J was only able to add the value of column (J+1) to the contents of column J. Column N could only hold a constant, and column 1 would contain the current result of the calculation. The initial condition was that column 1 was set to the initial value X of the polynomial, column 2 to a value derived from the first and higher derivatives of the polynomial at the same value of X, and columns 3 to N to values derived from the first and higher derivatives of the polynomial.

Babbage used the example $f(X) = X^2 + X + 41$ when explaining these principles, illustrating it using Table 1. The 2nd difference (analogous to 2nd derivative) is constant for a quadratic function. The rest of the table can be calculated using only addition. For example, in the last steps, 2 is added to 10 to give 12 (the 1st difference), then 12 is added to 71 to give the final result of $f(X)$ = 83. The previous steps are similarly calculated.

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X	f(X)	1st	2nd
0	41		
		2	
1	43		2
		2	
2	47		2
		6	
3	53		2
		8	
4	61		2
		10	
5	71		2
		12	
6	83		

Table 1 *Charles Babbage's example difference engine calculation*

Babbage's difference engines were entirely mechanical, but never completed. The later reconstruction of difference engine no.2 at the London Science Museum under the guidance of Doron Swade [15] (see Figure 1) demonstrated that it was able to be constructed with the technology of the 1850s. Smaller scale demonstration models have been constructed in Lego [19] and Meccano [20], as well as educational software models, e.g [21] and [24].

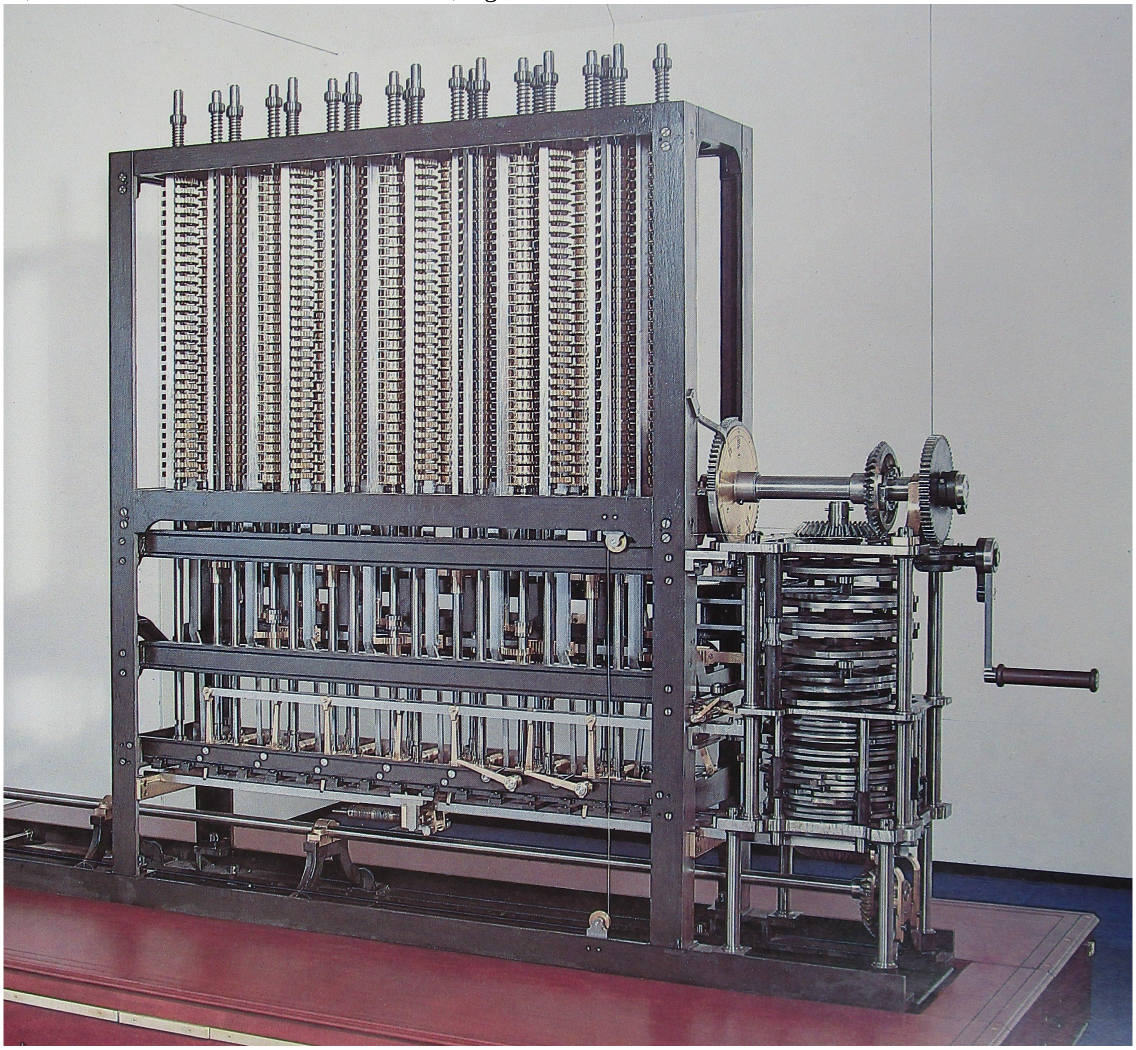

Figure 1 *Charles Babbage's Difference Engine No.2 in the Science Museum, London*
Completed in 2002, with 8,000 parts, 11 feet long, and weighing five tons,

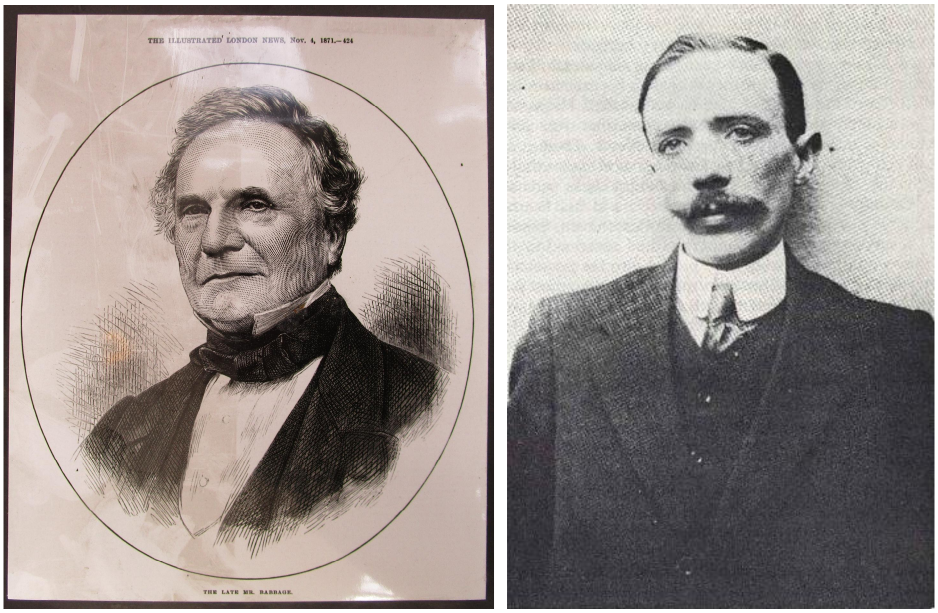

Figure 2 *(a) Charles Babbage, from his Obituary, 4th November 1871, The Illustrated London News*
(b) Percy Ludgate, image reproduced courtesy Brian Randell

II.2. Ludgate's Difference Engine

Percy Ludgate is shown in Figure 2(b), probably taken in the last five years of his life. He was the second person in history to publish (in 1909) a design for an analytical machine. But later, in 1914, he published a paper in the Handbook of the Napier Tercentenary Exhibition [3] in which he stated that he had designed a simple difference engine. From Ludgate 1914:

> The most pleasing characteristic of a difference-engine made on Babbage's principle is the simplicity of its action, the differences being added together in unvarying sequence; but notwithstanding its simple action, its structure is complicated by a large amount of adding mechanism—a complete set of adding wheels with carrying gear being required for the tabular number, and every order of difference except the highest order. On the other hand, while the best feature of the analytical engine or machine is the Jacquard apparatus (which, without being itself complicated, may be made a powerful instrument for interpreting mathematical formulae), its weakness lies in the diversity of movements the Jacquard apparatus must control. Impressed by these facts, and with the desirability of reducing the expense of construction, I designed a second machine in which are combined the best principles of both the analytical and difference types, and from which are excluded their more expensive characteristics. By using a Jacquard I found it possible to eliminate the redundancy of parts hitherto found in difference-engines, while retaining the native symmetry of structure and harmony of action of machines of that class. My second machine, of which the design is on the point of completion, will contain but one set of adding wheels, and its movements will have a rhythm resembling that of the Jacquard loom itself. It is primarily intended to be used as a difference-machine, the number of orders of differences being sixteen. Moreover, the machine will also have the power of automatically evaluating a wide range of miscellaneous formulae.

Ludgate 1914 only mentions the "Jacquard apparatus" (sequencer) and "one set of adding wheels" (accumulator). There is no mention of the novel storage or multiplier mechanisms of his analytical machine, so one might reasonably presume they were absent. The implication is that Ludgate traded physical complexity for programming, another novel Ludgate idea, and one that predated reduced instruction set computers (RISC) by eighty years.

It is clear the engine adopted an accumulator architecture. One might reasonably presume his accumulator could do subtraction by reverse rotation of the adding wheels, as did the "Mill" of his analytical machine. But there is no mention of any multiplication and division.

Jacquard looms could not do conditional operations, and neither could Babbage's difference engines (difference engines don't need looping constructs). Ludgate 1914 doesn't preclude his design being in essence an analytical engine behaving like a difference engine, but neither does it encourage it. The simpler Ludgate made the engine, the faster it could cycle (a la RISC machines), trading physical complexity not only for programming but also for speed. If there was no conditionality (no testing of conditions), and only ripple carry (2 x slower ripple carry but maybe 2 x faster cycling), it might avoid the penalties associated with ripple carry.

Babbage's difference engine no.2 was hand-cranked, with four turns of the crank per machine cycle. Each cycle consisted of a sequence of 4 simultaneous 31-digit additions, 4 ripple carry corrections, 3 simultaneous 31-digit additions, and 3 ripple carry corrections, i.e. 7 additions with carries excluding print and typeset operations. It takes about 8 seconds per cycle [15]. By way of comparison, Ludgate's analytical engine appears to have had a cycle time of $\frac{1}{3}$ of a second, with each Mill wheel counting as fast as one step per $\frac{1}{100}$ of a second. If Ludgate's difference engine only performed addition and subtraction, then its accumulator wheels could count at this rate, allowing an addition time of about $\frac{1}{10}$ of a second without ripple carry, or perhaps $\frac{1}{5}$ of a second with ripple carry. Therefore the 7 additions with carries would take about 1.2 seconds, faster but not very much faster than Babbage's, but with a much simpler mechanism, and the ability to operate automatically from programs on Jacquard cards.

One big puzzle is whether there was proper storage. Ludgate stated there would be 16 orders of difference, so at minimum 16 temporary registers. One possibility would be a set of 16 registers constantly moving like a conveyor belt. Perhaps Ludgate poached from his analytical machine ideas of shuttles in storage cylinders, perhaps with shuttles in a constantly rotating ring, or with shuttles in a constantly rotating storage cylinder (with no address selection), somewhat reminiscent of Babbage's 1932 introduction of a "feedback mechanism" to his Difference Engine [25], the idea that ultimately appears to have spawned his Analytical Engine.

The kernel of the operations could be as in Algorithm 1.

```
repeat
    acc = initial_value
    for d in row_of_differences
        acc = acc + d
        d = acc
    print acc
```

Algorithm 1 *Speculative example operations of Ludgate's difference engine calculation*

So to calculate the squares one might initialize with row of differences = [2, -1, 0], yielding a printout of 1, 4, 9, 16. The temporary storage for `row_of_differences` would not do any calculation, but it would need to present its contents to the accumulator in such a way that the contents were added to it, and then be set to the value of the accumulator.

With 16 orders of difference it might be useful to scale (shift towards most or least significant digit) the accumulator at each iteration [23], and/or to employ fixed-point arithmetic. Doron Swade [18] has said "The position of the decimal in the Difference Engines is notional, i.e. there is nothing mechanical that marks its location. In the case of DE2, which has vertical columns for multidigit numbers, the position of the decimal point is along a notional fixed horizontal line across the columns." This approach is equally applicable to Ludgate's difference engine.

With no ordinals nor logarithmic indexes, temporary storage shuttles could hold decimals on rods, avoiding much of the complexity of Ludgate's analytical machine. Furthermore, if the temporary storage held quantities 0..10 for each difference rather than 0..9, then additions could give 20 with just a single carry, rather than two rounds of carry. This would avoid ripple carry until the very end, and virtually double the speed.

Another puzzle is what exactly the program steps might execute. An initial instruction might carry the initial value of $f(X)$ and the differences (with at minimum just the 1st difference and the rest zero), perhaps with these operands in succeeding rows of the Jacquard cards, until a hole marked the last operand. Subsequent instructions might perform the requisite additions or subtractions until the calculation was complete, followed by printing of the result. À la unrolled iterations, further in-line instructions could repeat this pattern of arithmetic and printing instructions until a whole table had been calculated and printed. The nett outcome would be a programmable difference engine. The Jacquard cards would be consumed in one direction with no looping, just as in Jacquard looms. From Ludgate 1914: "its movements will have a rhythm resembling that of the Jacquard loom itself", which conjures up an image of the engine merrily gobbling up the Jacquard cards.

Ludgate's final two sentences are intriguing:

> It is primarily intended to be used as a difference-machine, the number of orders of differences being sixteen. Moreover, the machine will also have the power of automatically evaluating a wide range of miscellaneous formulae.

Randell has highlighted that the words "It is primarily intended" indicate that Ludgate had a wider view of its powers, that it was not strictly a difference engine, that it could do more than a pure difference engine could. But Ludgate's wording does not hint at any addressible storage, nor conditional branching. If they were absent, then it was just a special-purpose machine (and faster without conditional branching), a sort of "formulae evaluation engine", certainly not an analytical engine. An analytical engine can be programmed to execute any of its instruction set including conditional instructions in any order (not a fixed set of operations in a fixed order), and has addressible storage with programmable load/store. A pure difference engine cannot do any of those operations, but perhaps the machine mentioned in Ludgate 1914 could do some subset of those operations. Lack of hints does not mean absence, we cannot assume, we can only speculate

III. Concluding Remarks

The above must not be construed as anything but speculations. The paucity of information in Ludgate 1914 precludes any other complexion. What Ludgate states does, however, imply an entirely novel concept at the time.

IV. Acknowledgements

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