Calculating History

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Jacobi indices

This page is an appendix to "Irish Logarithms Part 2"

The Jacobi index Ind(z) of an integer z can be given by

$$z = g^{\mathrm{Ind}(z)} \text{ modulo } p \qquad (1)$$

The trick is to find values of g and p for which this equation applies and for which Ind(z) is unique for each value of z in a selected set of numbers. If p is prime and the set of z's consists of all integers between 1 and $p - 1$, then there is a g for which each z has an unique index between 0 and $p - 2$. If so, then:

$$\mathrm{Ind}(z_1 \times z_2) = \mathrm{Ind}(z_1) + \mathrm{Ind}(z_2) \qquad (2)$$

So using Ind(), we can multiply by adding.[1]

For a calculator like Verea's, we only need the indices of the 36 integers in the simple multiplication table.

So for a calculator the set of z's differs from the set of all integer between 1 and $p - 1$: there are "gaps" in the collection and the collection does not end with a prime ($p - 1 = 81$).

Because in the calculator only numbers less than 10 are multiplied, equation (2) does not have to apply for all z_1 and z_2, but only for z_1, z_2 <10. The resulting index Ind($z_1 \times z_2$) must be unique for all unique simple products.

So we do not have to apply the Jacobi indices literally for the multiplication table of a calculating machine. We can still try using equation (1) to generate indices that meet our goal. There is no guarantee that the indices that we find are the **smallest** numbers causing Ind($z_1 \times z_2$) = Ind(z_1) + Ind(z_2). Our final goal is to minimize the largest index.

Using equation (1) with $p = 11$ and $g = 2$, I found two sets of indices with the largest index less than 100. The following table shows the indices for the integers <10:

z	1	2	3	4	5	6	7	8	9

Ind(z)	0	1	18	2	44	19	7	4	36
Ind(z)	0	1	8	2	44	9	27	4	16

Other choices of p and g may provide better (i.e. lower) indices.

The indices of Schumacher's slide rule are generated with $p = 101$ and $g = 2$.

Notes

1. Strictly speaking: the sum of the indices modulo $p - 1$.

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