AccessionIndex: TCD-SCSS-V.20121208.852
Accession Date: 8-Dec-2012
Accession By: Prof.J.G.Byrne

Object name: Posters on 'The Tercentenary of π - 1706-2006'
Vintage: c.2006
Synopsis: Trinity College Dublin, for an exhibition in 2006 to celebrate the 300th anniversary of π.

Description:
Assorted illustrations and typed captions mounted on cardboard, evidently used for the 'Tercentenary of π - 1706-2006' exhibition held in Trinity College Dublin in 2006 celebrating the 300th anniversary of π.

These exhibits were prepared by Prof.J.G.Byrne of the Dept.Computer Science, Trinity College Dublin.

The homepage for this catalog is at: https://www.scss.tcd.ie/SCSSTreasuresCatalog/
Click *'Accession Index'* (1st column listed) for related folder, or *'About'* for further guidance. Some of the items below may be more properly part of other categories of this catalog, but are listed here for convenience.

Accession Index	Object with Identification
TCD-SCSS-V.20121208.852	Posters on 'The Tercentenary of π - 1706-2006', Trinity College Dublin, for an exhibition in 2006 to celebrate the 300th anniversary of π, c.2006. *[Prof.J.G.Byrne's Tercentenary of π exhibits]*
TCD-SCSS-V.20121208.850	Poster advertising 'The Tercentenary of π - 1706-2006', Trinity College Dublin, for exhibition celebrating 300th anniversary of π, c.2006.
TCD-SCSS-V.20121208.851	Poster of graphic based upon Archimedes spiral, Gerry O'Brien, Dept.Computer Science, Trinity College Dublin, for the 'Tercentenary of π - 1706-2006' exhibition, c.2006.

References:

1. Wikipedia, *Pi*, see:.
https://en.wikipedia.org/wiki/Pi
Last browsed to: 16-Feb-2018.

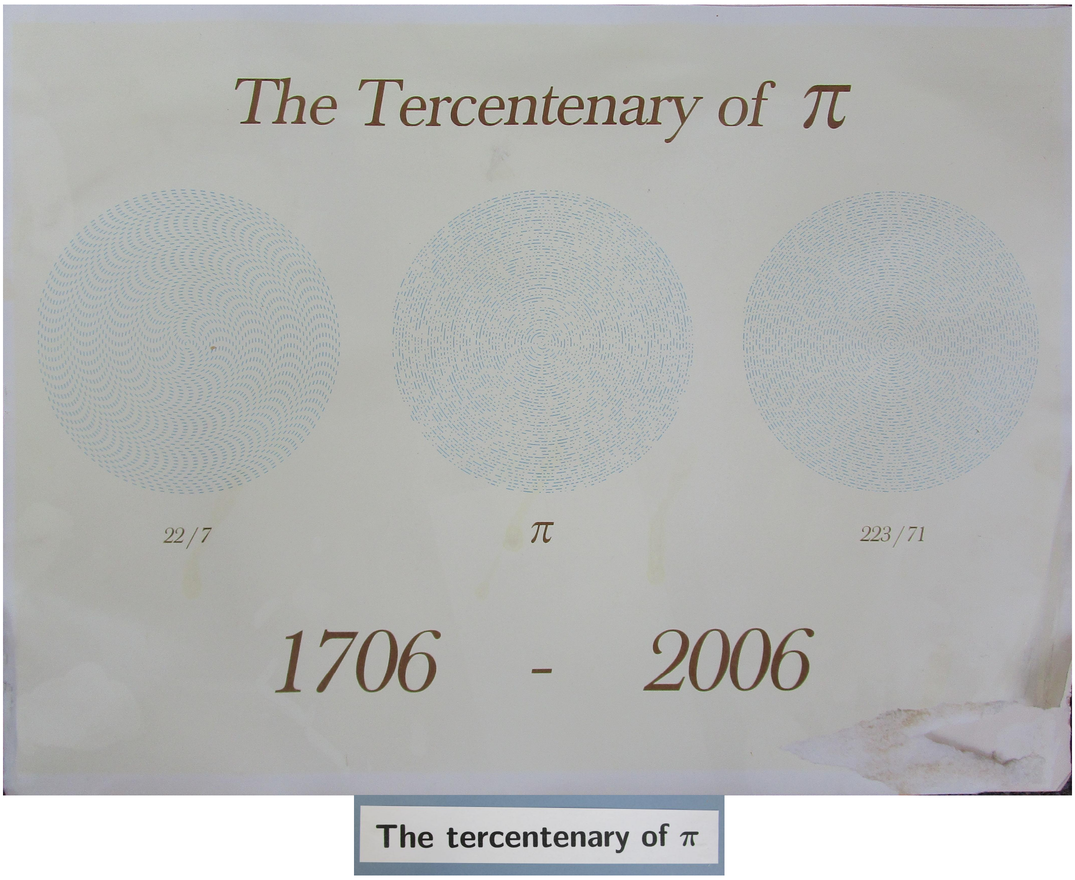


Figure 1: The Tercentenary of π

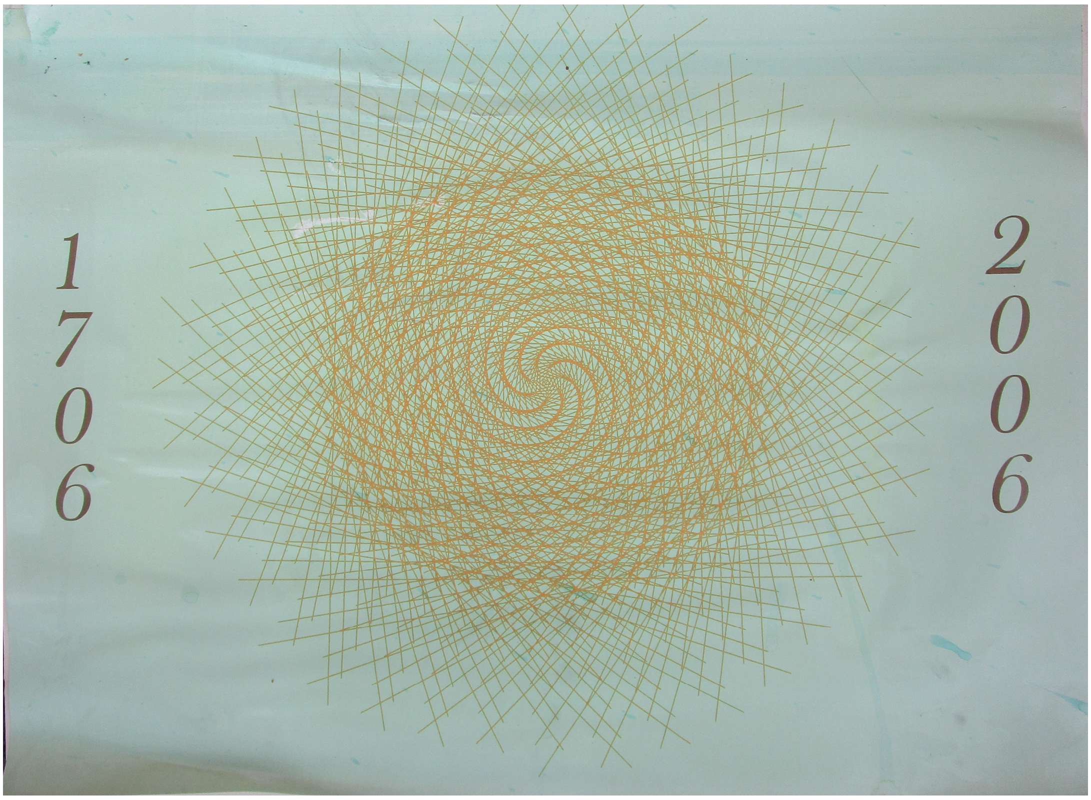
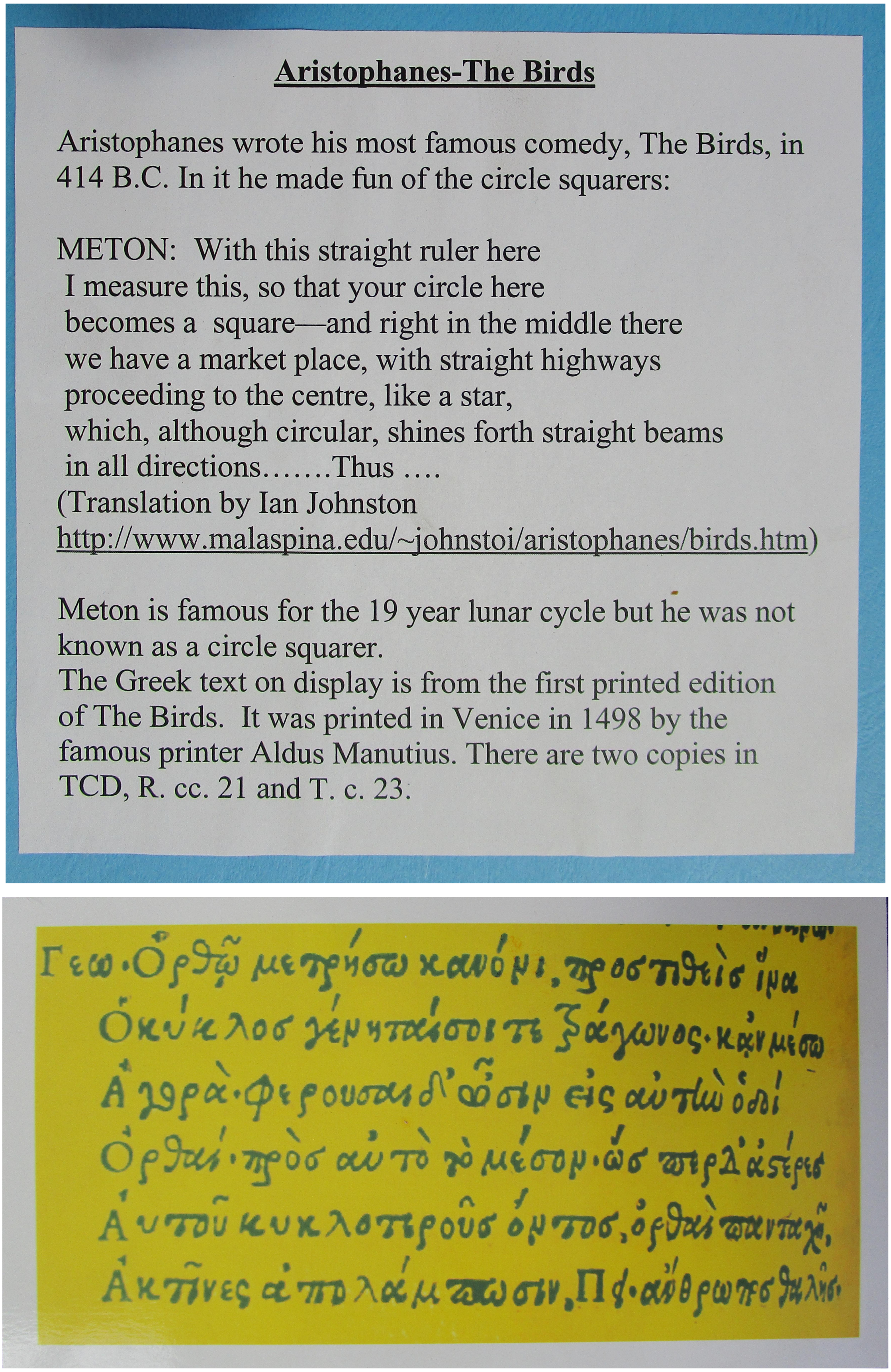


Figure 2: Poster of graphic based upon Archimedes spiral created by Gerry O'Brien, Dept.Computer Science, Trinity College Dublin, for the 'Tercentenary of π ' exhibition

The circle squarers

One of the problems which intrigued the Greeks was that of squaring the circle, that is, using only a straight edge and compass, to draw a square exactly equal in area to a circle. The problem dates from at least the time of Anaxagoras of Clazomenae (499 B.C.-428 B.C.) who amused himself trying to do it while in prison. Aristophanes made fun of circle squarers in his comedy The Birds (414 B.C.). Although it was not proved impossible until 1882 it was long suspected that it was not possible. The variety of people who tried to do it is truly amazing. These range from serious and respected mathematicians such as Cardinal Nicolas of Cusa (EE. ee. 30. No. 4.), the Jesuit Gregory of Saint Vincent (L. bb. 15.), whose work was highly praised by Leibniz, and Orontius Fineus (L. bb. 23. No. 3.), a professor of mathematics, to people distinguished in other areas such as the philosopher Thomas Hobbes (L. ll. 14.) and the distinguished polymath Joseph Scaliger (L. cc. 9. No. 1). Both of these were hopelessly wrong about their value of π but could not be convinced of their error. Many others were just cranks. An example is James Smith (e. g. Gall. FF. 6. 91. No. 5), a Liverpool businessman, who was convinced that $\pi = 3.125$, despite Sir William Rowan Hamilton showing that it had to be greater than this value.

Figure 3: c.499BC: Circle-squarers



Aristophanes-The Birds

Aristophanes wrote his most famous comedy, The Birds, in 414 B.C. In it he made fun of the circle squarers:

METON: With this straight ruler here
I measure this, so that your circle here
becomes a square—and right in the middle there
we have a market place, with straight highways
proceeding to the centre, like a star,
which, although circular, shines forth straight beams
in all directions…….Thus ….
(Translation by Ian Johnston
http://www.malaspina.edu/~johnstoi/aristophanes/birds.htm)

Meton is famous for the 19 year lunar cycle but he was not known as a circle squarer.
The Greek text on display is from the first printed edition of The Birds. It was printed in Venice in 1498 by the famous printer Aldus Manutius. There are two copies in TCD, R. cc. 21 and T. c. 23.

Γεω. Ὀρθῷ μετρήσω κανόνι, προστιθεὶς ἵνα
ὁ κύκλος γένηταί σοι τετράγωνος. κἀν μέσῳ
ἀγορά. φέρουσαι δ' ὦσιν εἰς αὐτὴν ὁδοὶ
ὀρθαὶ πρὸς αὐτὸ τὸ μέσον. ὥσπερ δ' ἀστέρος
αὐτοῦ κυκλοτεροῦς ὄντος, ὀρθαὶ πανταχῇ
ἀκτῖνες ἀπολάμπωσιν. Πισ. ἅνθρωπος Θαλῆς.

Figure 4: c.414BC: Aristophanes 'The Birds'

Figure 5: c.150: Ptolemy Almagest, a treatise on the apparent motions of the stars and planetary paths

First, then, [see Fig. 1.1] let there be a semi-circle ABG about centre D and on diameter ADG. Draw DB perpendicular to AG at D. Let DG be bisected at E, join EB, and let EZ be made equal to EB. Join ZB.

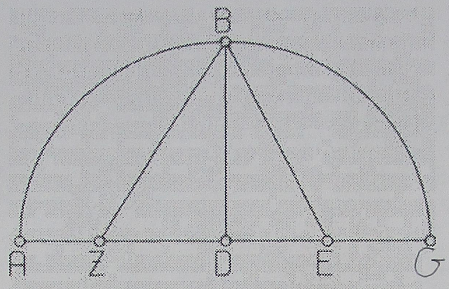


Fig. 1.1

I say that ZD is the side of the [regular] decagon, and BZ the side of the [regular] pentagon.

[Proof:] Since the straight line DG is bisected at E, and a straight line DZ is adjacent to it,

$GZ.ZD + ED^2 = EZ^2$.
But $EZ^2 = BE^2$ (EB = ZE),
and $EB^2 = ED^2 + DB^2$.
$\therefore GZ.ZD + ED^2 = ED^2 + DB^2$.
$\therefore$ $GZ.ZD = DB^2$ (subtracting ED^2, common).
$\therefore$ $GZ.ZD = DG^2$

So Z has been cut in extreme and mean ratio at D. Now since the side of the hexagon and the side of the decagon, when both are inscribed in the same circle, make up the extreme and mean ratios of the same straight line, and since GD, being a radius, represents the side of the hexagon DZ is equal to the side of the decagon.
(From G. J. Toomer's translation of Ptolemy's Almagest, London:Duckworth 1984)

Figure 6: c.150: Ptolemy Almagest

Joannes Buteo
(1492-1572)

Buteo's *De quadratura circuli* (L. oo. 34) was probably the first survey of values of π obtained by many people. He was a strong supporter of Archimedes. The diagram indicates whether the values were too low or too high, with respect to Archimedes bounds. He showed that many claims to squaring the circle were false including those of his teacher Orontius Fineus, a professor of mathematics.

Figure 7: c.1492-1572: Buteo

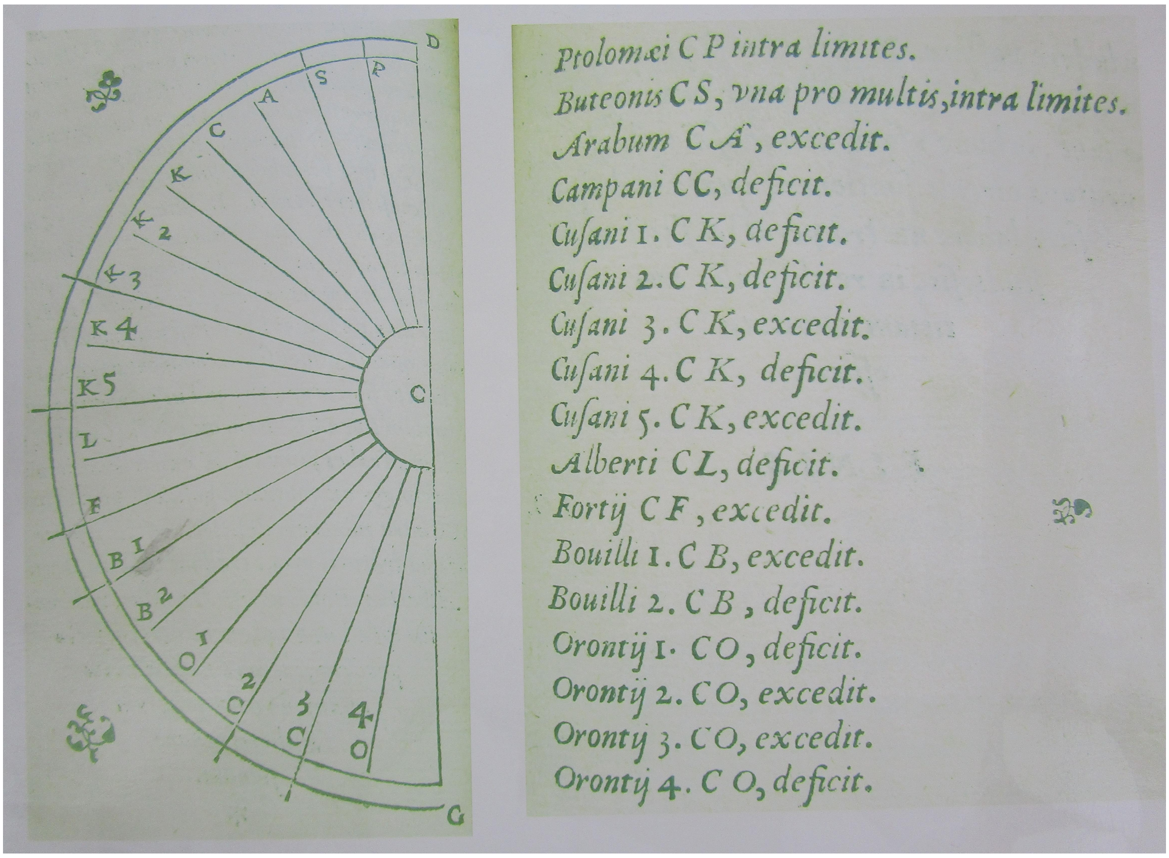
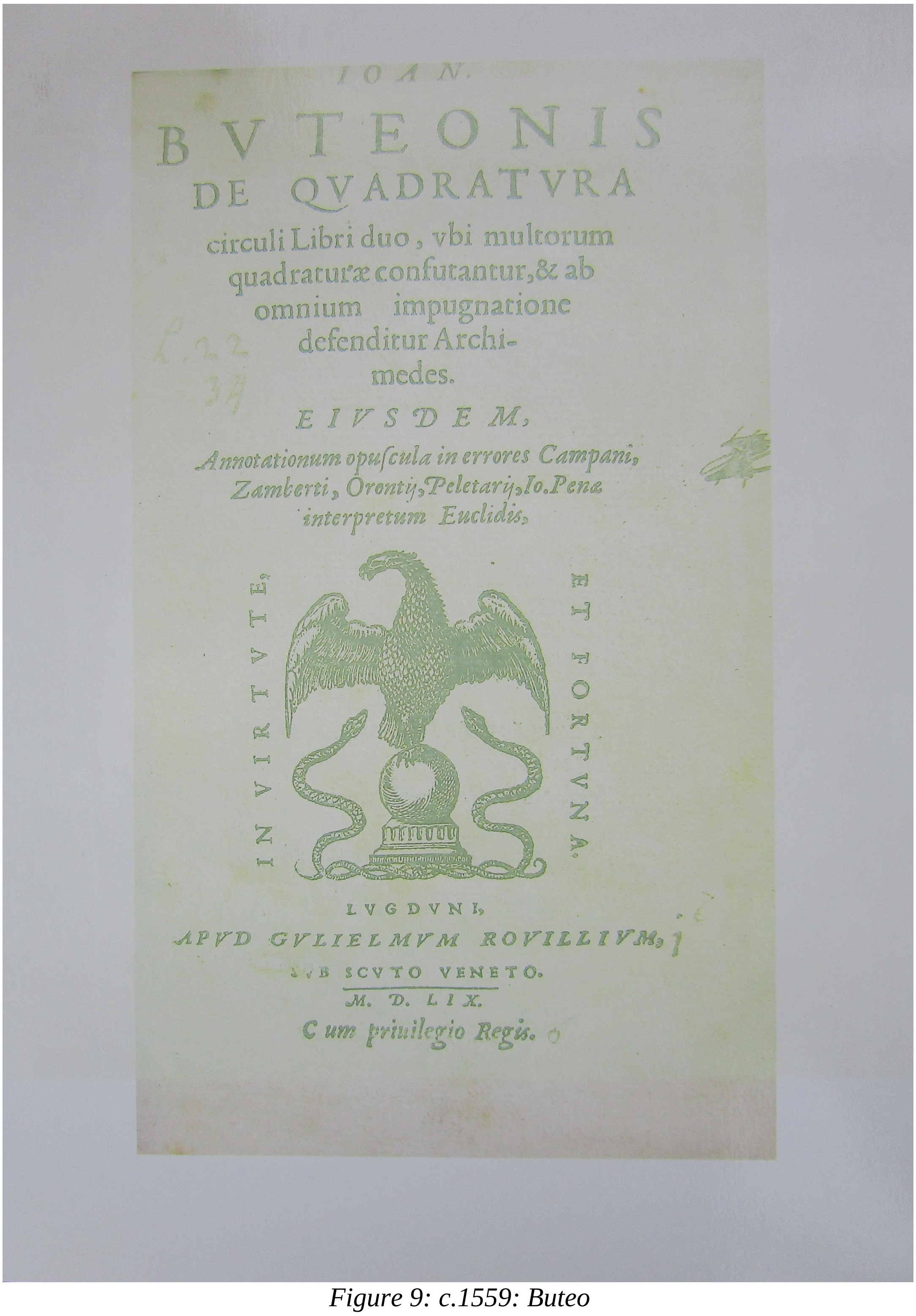


Figure 8: c.1559: Buteo


IOAN.

BVTEONIS

DE QVADRATVRA

circuli Libri duo, vbi multorum quadraturæ confutantur, & ab omnium impugnatione defenditur Archimedes.

EIVSDEM,

Annotationum opuscula in errores Campani, Zamberti, Orontij, Peletarij, Io. Penæ interpretum Euclidis.

IN VIRTVTE, ET FORTVNA.

LVGDVNI,

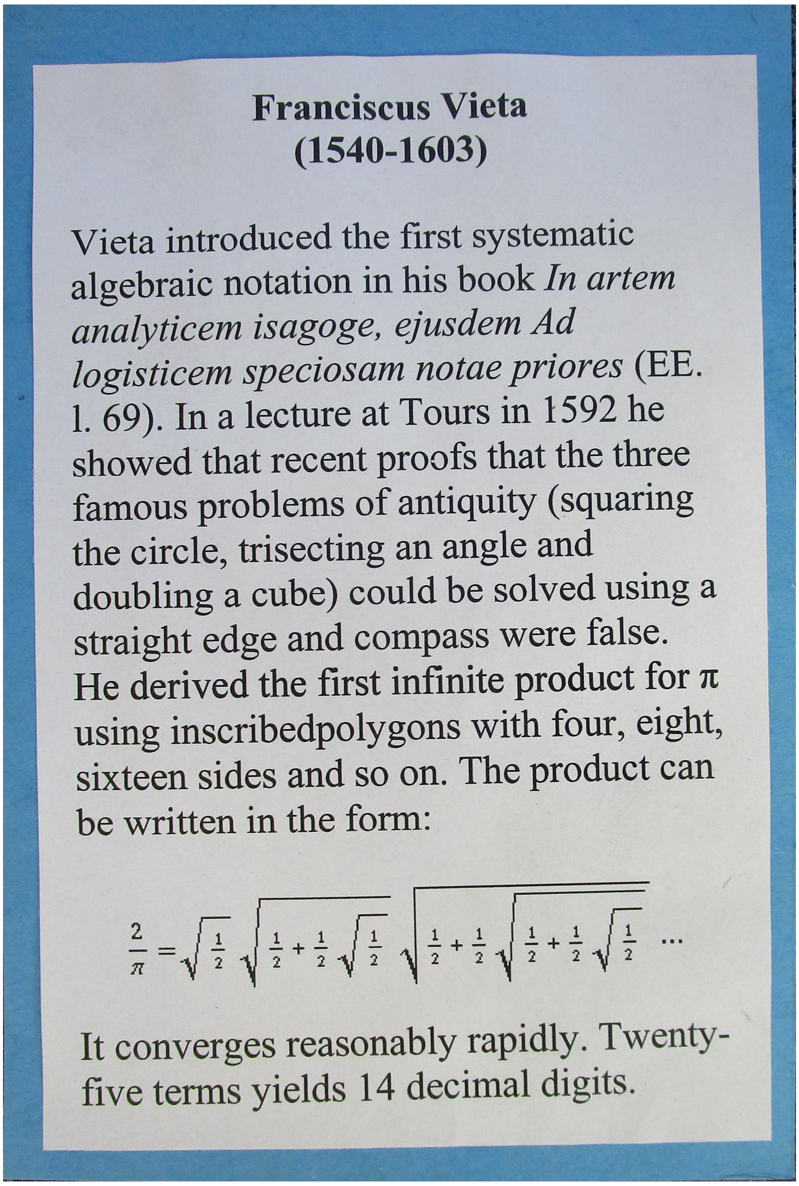
APVD GVLIELMVM ROVILLIVM,

SVB SCVTO VENETO.

M. D. LIX.

Cum priuilegio Regis.

Figure 9: c.1559: Buteo



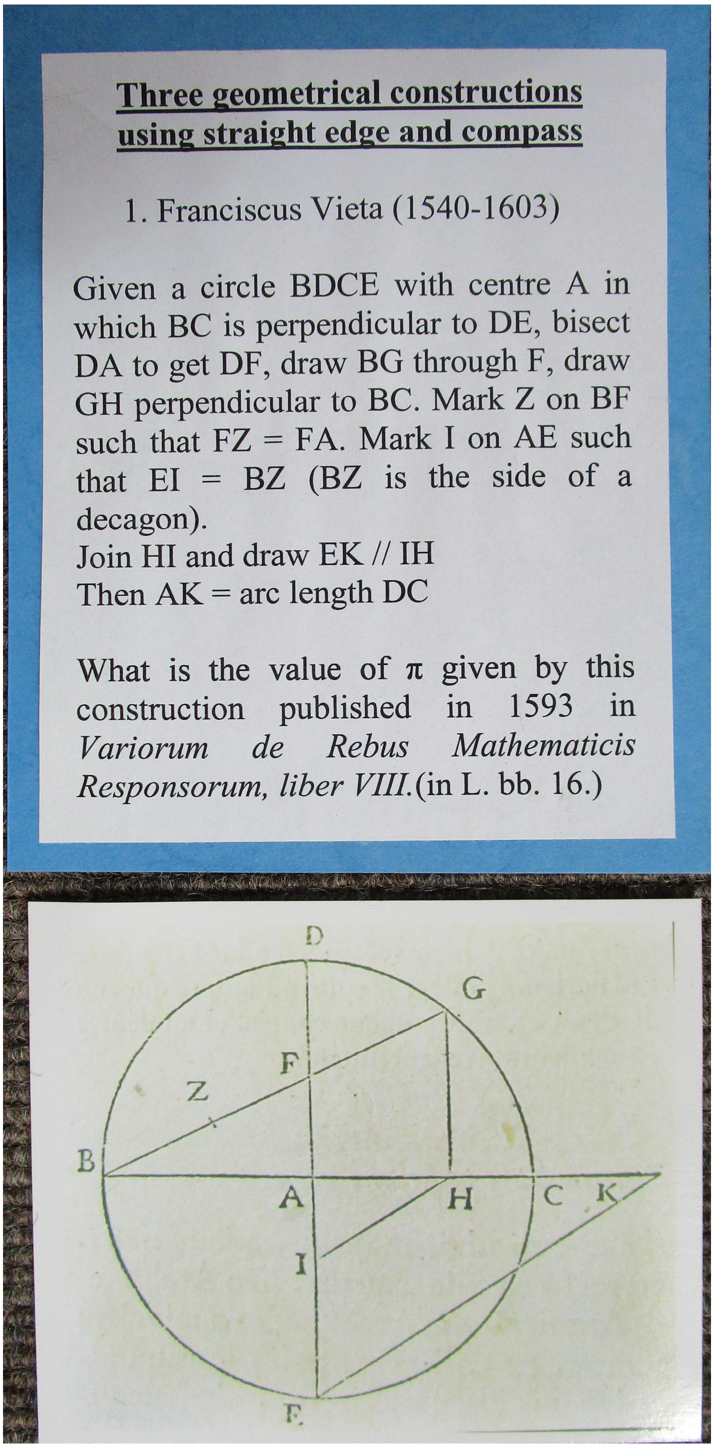
Francisus Vieta
(1540-1603)

Vieta introduced the first systematic algebraic notation in his book *In artem analyticem isagoge, ejusdem Ad logisticem speciosam notae priores* (EE. I. 69). In a lecture at Tours in 1592 he showed that recent proofs that the three famous problems of antiquity (squaring the circle, trisecting an angle and doubling a cube) could be solved using a straight edge and compass were false. He derived the first infinite product for π using inscribedpolygons with four, eight, sixteen sides and so on. The product can be written in the form:

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\cdots$$

It converges reasonably rapidly. Twenty-five terms yields 14 decimal digits.

Figure 10: c.1592: Vieta



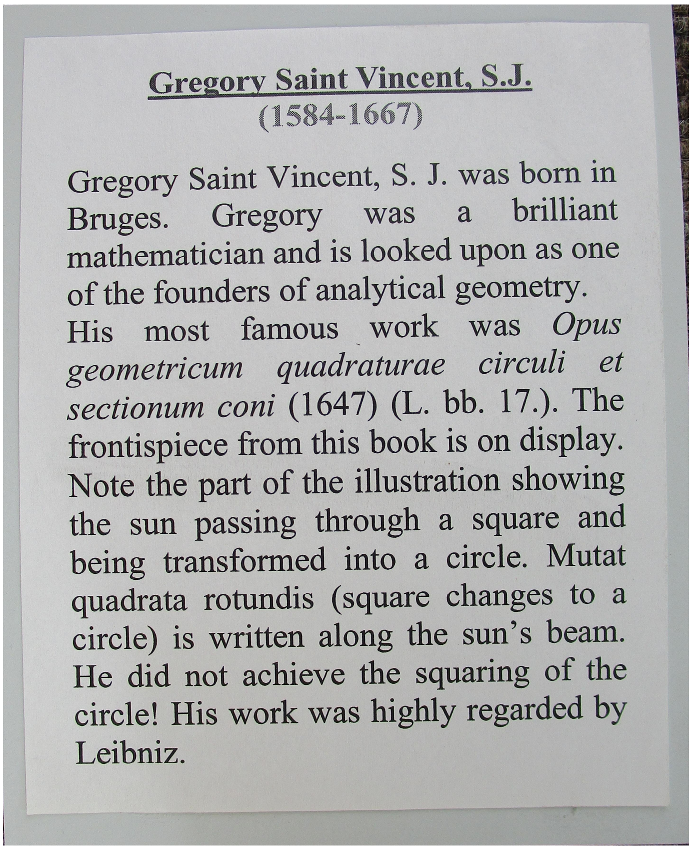
Three geometrical constructions using straight edge and compass

1. Francis
cus Vieta (1540-1603)

Given a circle BDCE with centre A in which BC is perpendicular to DE, bisect DA to get DF, draw BG through F, draw GH perpendicular to BC. Mark Z on BF such that FZ = FA. Mark I on AE such that EI = BZ (BZ is the side of a decagon).
Join HI and draw EK // IH
Then AK = arc length DC

What is the value of π given by this construction published in 1593 in *Variorum de Rebus Mathematicis Responsorum, liber VIII.*(in L. bb. 16.)

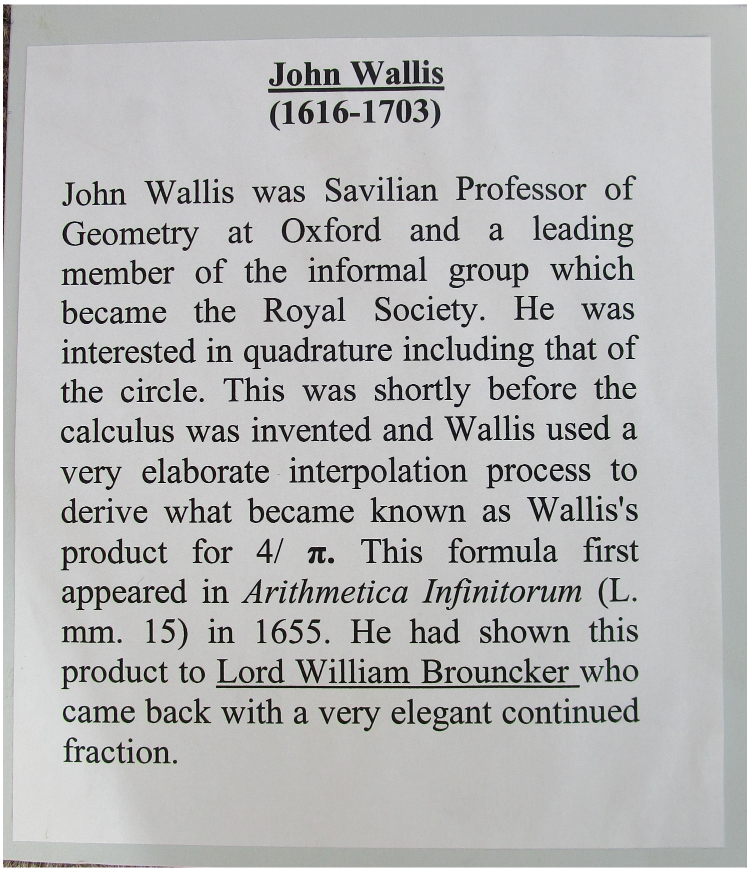
Figure 11: c.1593: Vieta



Gregory Saint Vincent, S.J.
(1584-1667)

Gregory Saint Vincent, S. J. was born in Bruges. Gregory was a brilliant mathematician and is looked upon as one of the founders of analytical geometry.
His most famous work was *Opus geometricum quadraturae circuli et sectionum coni* (1647) (L. bb. 17.). The frontispiece from this book is on display. Note the part of the illustration showing the sun passing through a square and being transformed into a circle. Mutat quadrata rotundis (square changes to a circle) is written along the sun's beam. He did not achieve the squaring of the circle! His work was highly regarded by Leibniz.

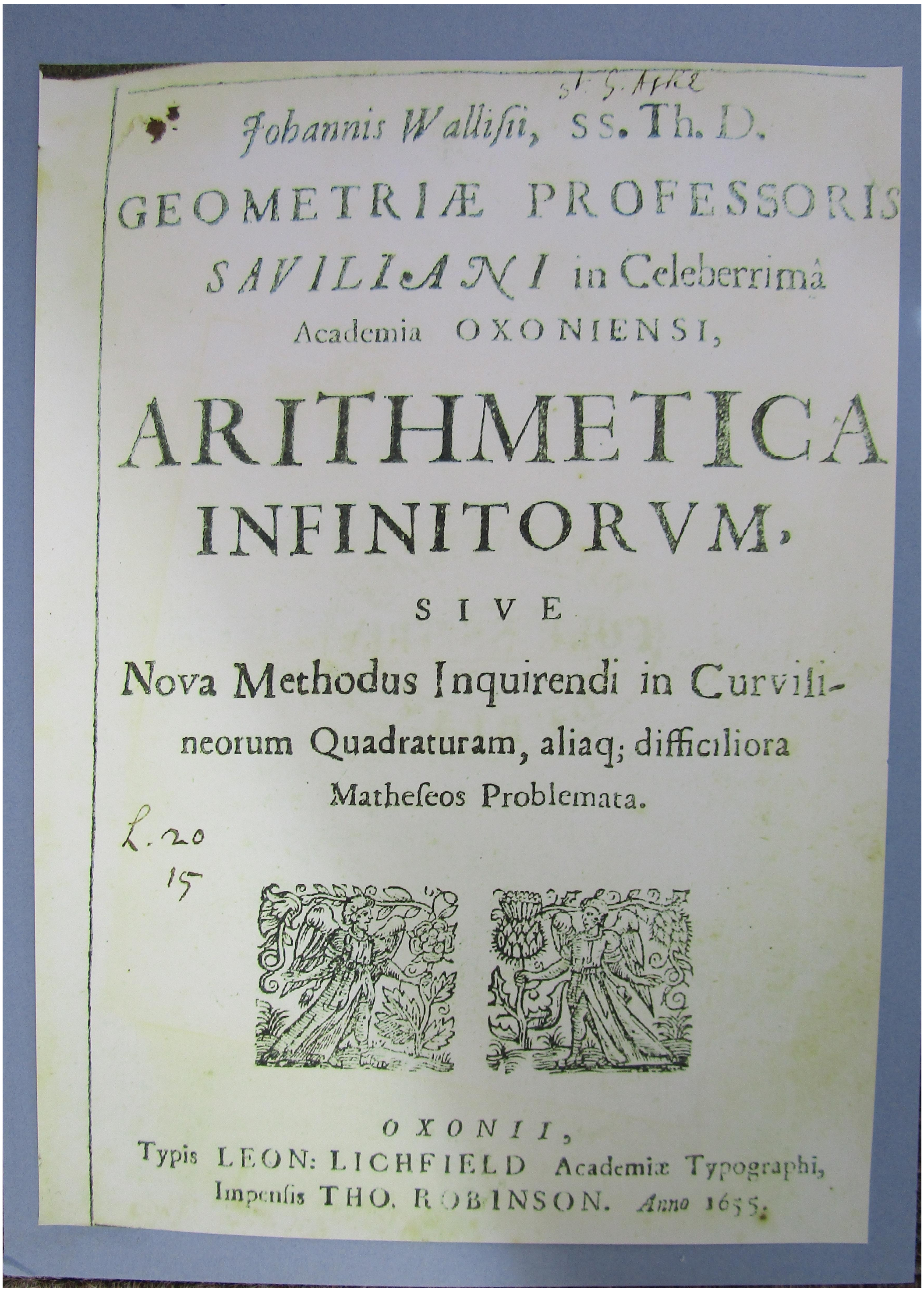
Figure 12: c.1647: Gregory Saint Vincent



John Wallis
(1616-1703)

John Wallis was Savilian Professor of Geometry at Oxford and a leading member of the informal group which became the Royal Society. He was interested in quadrature including that of the circle. This was shortly before the calculus was invented and Wallis used a very elaborate interpolation process to derive what became known as Wallis's product for 4/ **π.** This formula first appeared in *Arithmetica Infinitorum* (L. mm. 15) in 1655. He had shown this product to Lord William Brouncker who came back with a very elegant continued fraction.

Figure 13: c.1655: John Wallis



st. G. Apke

Johannis Wallisii, SS. Th. D.

GEOMETRIÆ PROFESSORIS

SAVILIANI in Celeberrimâ

Academia OXONIENSI,

ARITHMETICA INFINITORUM,

SIVE

Nova Methodus Inquirendi in Curvilineorum Quadraturam, aliaq; difficiliora Matheseos Problemata.

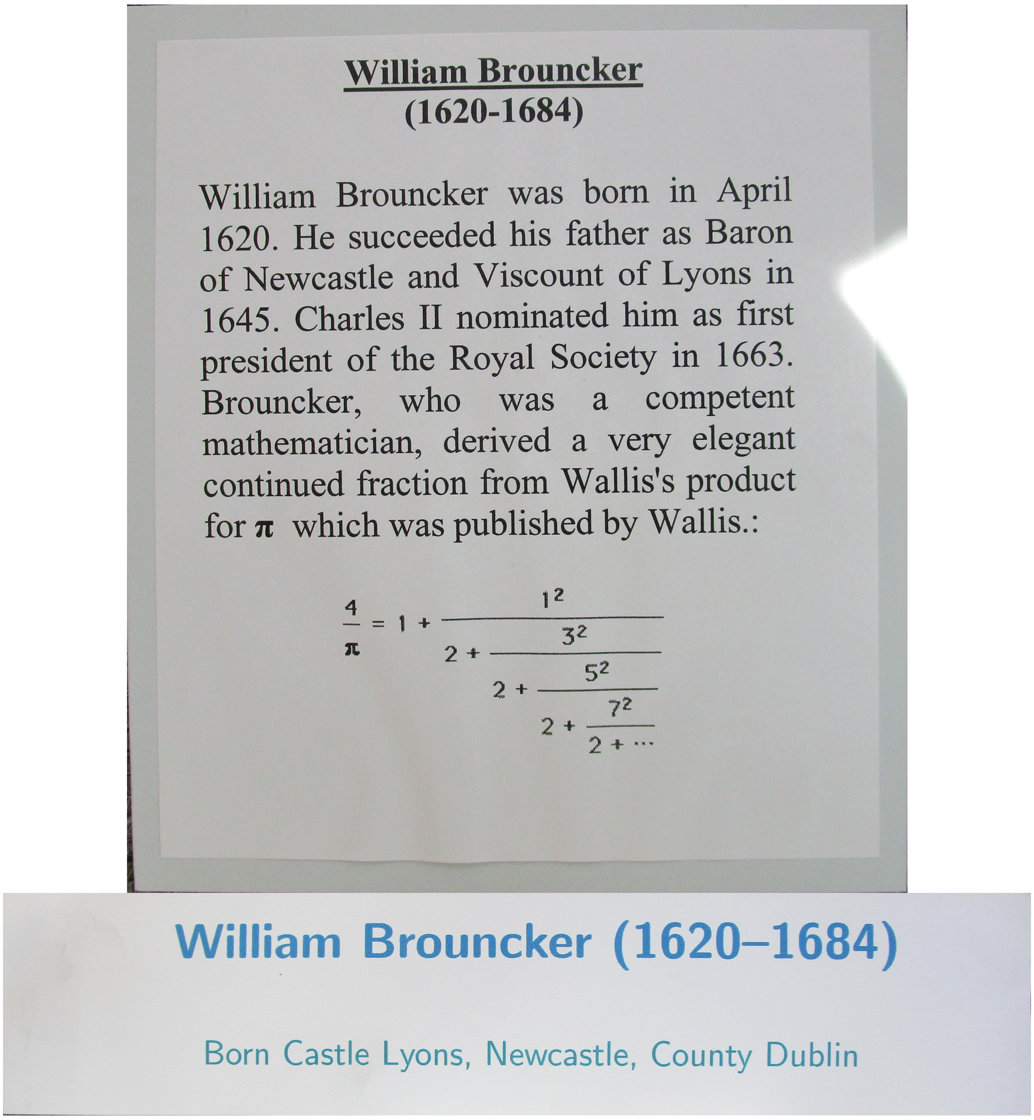
L. 20
15

OXONII,

Typis LEON: LICHFIELD Academiæ Typographi,

Impensis THO. ROBINSON. *Anno* 1655.

Figure 14: c.1655: John Wallis

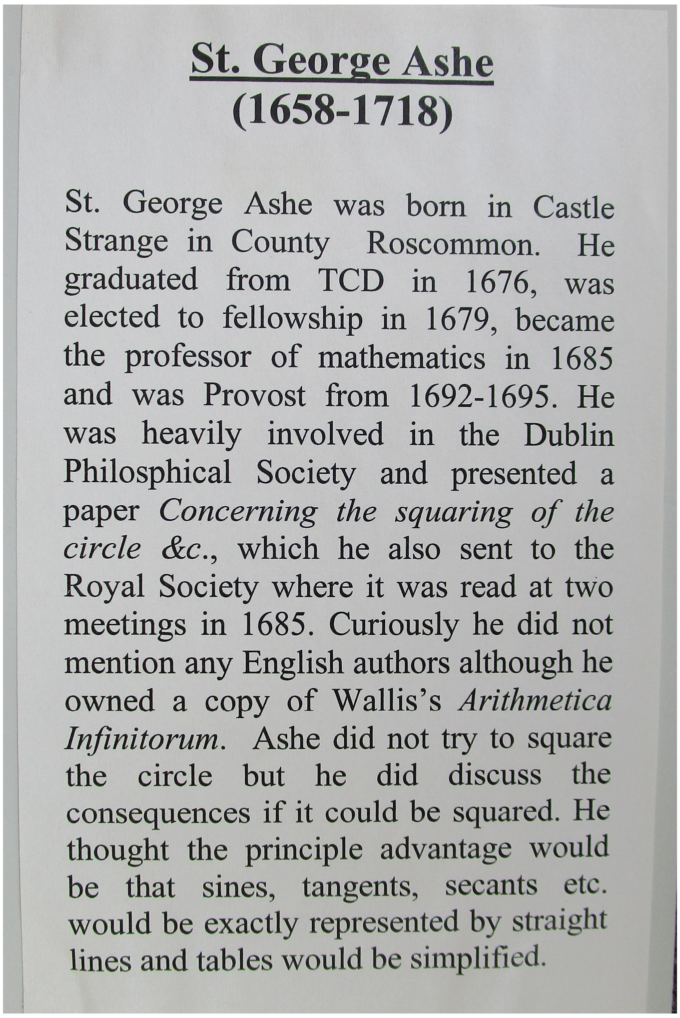


William Brouncker
(1620-1684)

William Brouncker was born in April 1620. He succeeded his father as Baron of Newcastle and Viscount of Lyons in 1645. Charles II nominated him as first president of the Royal Society in 1663. Brouncker, who was a competent mathematician, derived a very elegant continued fraction from Wallis's product for π which was published by Wallis.:

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cdots}}}}$$

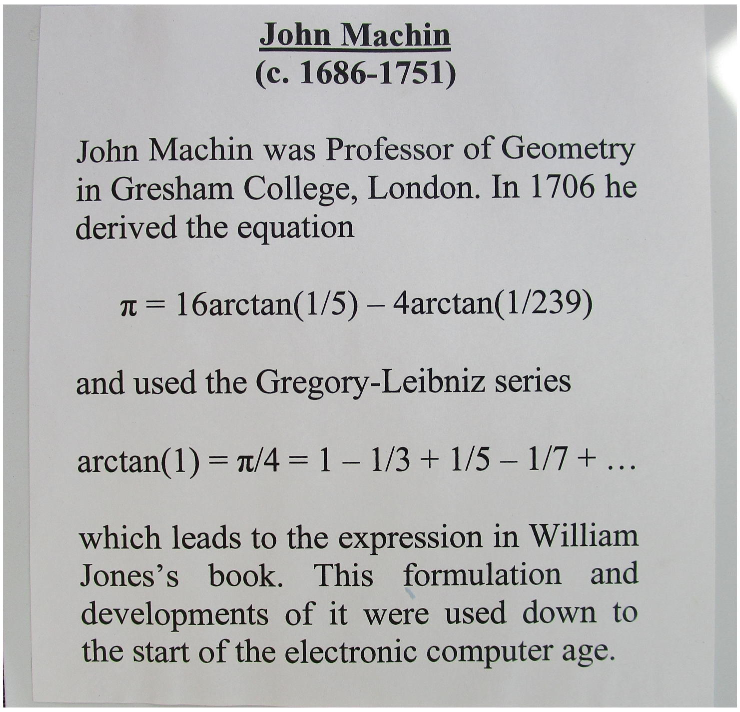
Figure 15: c.1663: William Brouncker



St. George Ashe
(1658-1718)

St. George Ashe was born in Castle Strange in County Roscommon. He graduated from TCD in 1676, was elected to fellowship in 1679, became the professor of mathematics in 1685 and was Provost from 1692-1695. He was heavily involved in the Dublin Philosphical Society and presented a paper *Concerning the squaring of the circle &c.*, which he also sent to the Royal Society where it was read at two meetings in 1685. Curiously he did not mention any English authors although he owned a copy of Wallis's *Arithmetica Infinitorum*. Ashe did not try to square the circle but he did discuss the consequences if it could be squared. He thought the principle advantage would be that sines, tangents, secants etc. would be exactly represented by straight lines and tables would be simplified.

Figure 16: c.1685: St.George Ashe



John Machin
(c. 1686-1751)

John Machin was Professor of Geometry in Gresham College, London. In 1706 he derived the equation

$$\pi = 16\arctan(1/5) - 4\arctan(1/239)$$

and used the Gregory-Leibniz series

$$\arctan(1) = \pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$$

which leads to the expression in William Jones's book. This formulation and developments of it were used down to the start of the electronic computer age.

Figure 17: c.1706: John Machin

COMMENTARII ACADEMIAE SCIENTIARVM IMPERIALIS PETROPOLITANAE.

TOMVS VII.

AD ANNOS cIↃIↃCCXXXIV. & cIↃIↃCCXXXV.

PETROPOLI,

TYPIS ACADEMIAE. cIↃIↃCCXL.

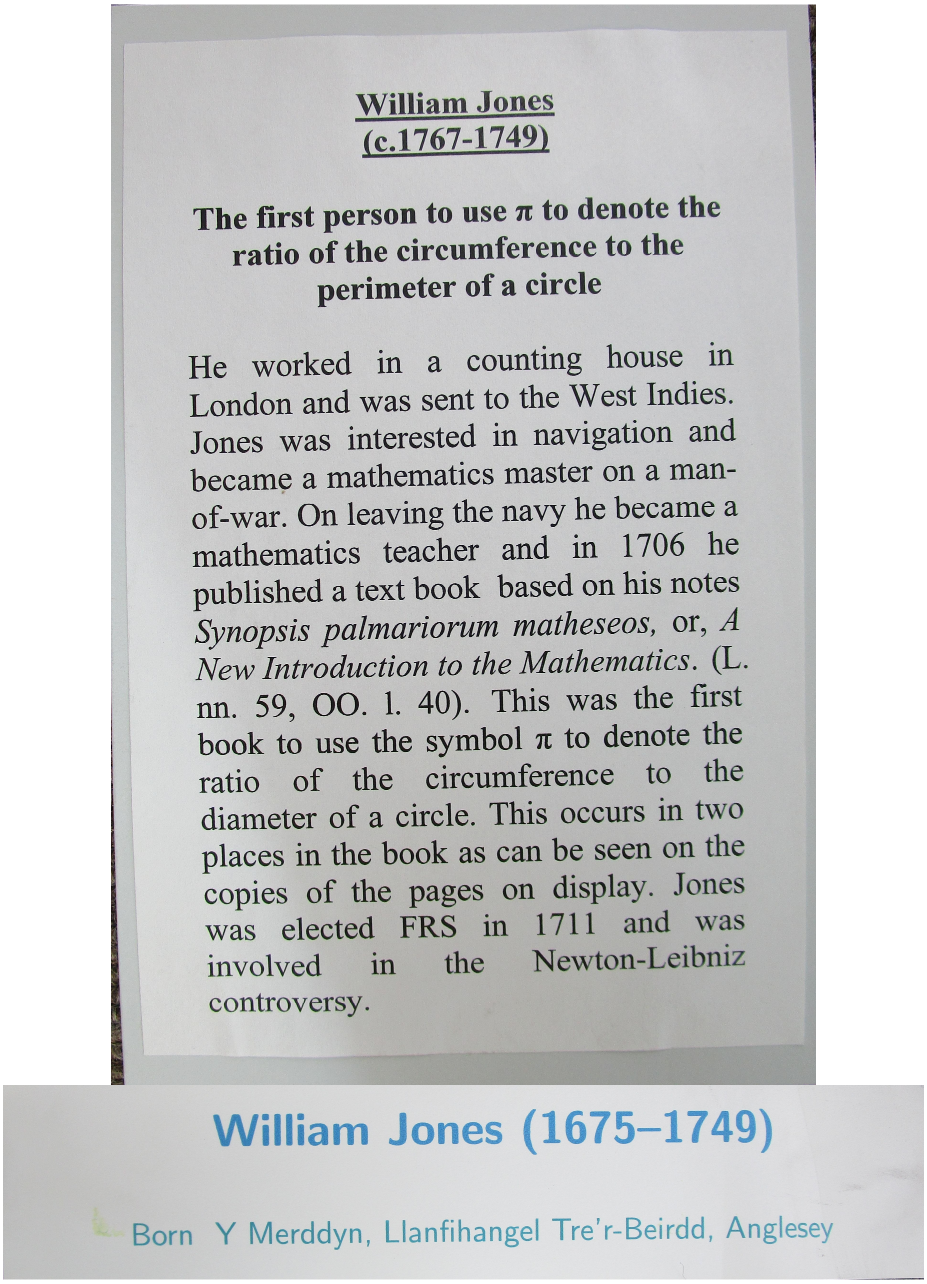
DE SVMMIS SERIERVM RECIPROCARVM.

AVCTORE

Leonh. Eulero.

$+\frac{1}{25}+\frac{1}{49}+$ etc. ſummam eſſe $=\frac{q^2}{2}=\frac{p^2}{8}$; denotante p totam circuli peripheriam, cuius diameter eſt $=1$. Summa autem huius ſeriei $1+\frac{1}{9}+\frac{1}{25}+$ etc. pendet a ſumma ſeriei $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+$ etc. quia haec quarta ſui parte minuta illam dat. Eſt ergo ſumma huius ſeriei aequalis ſummae illius cum ſui triente. Quamobrem erit $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}+$ etc. $=\frac{p^2}{6}$, ideoque huius ſeriei ſumma per 6 multiplicata aequalis eſt quadrato peripheriae circuli cuius diameter eſt 1; quae eſt ipſa propoſitio cuius initio mentionem feci.

Figure 18: c.1735: Euler Summis Serierum Reciprocarum

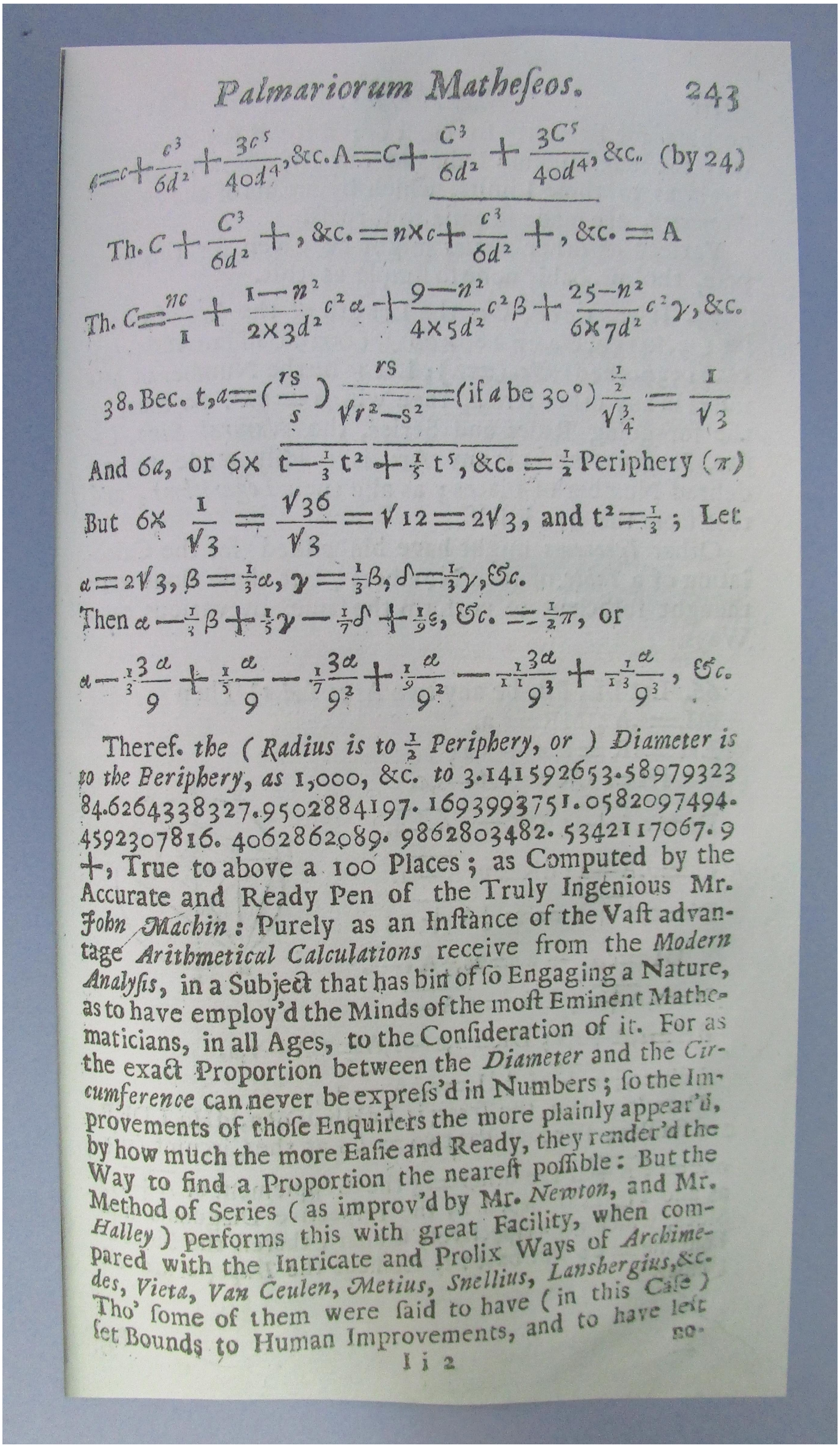


William Jones
(c.1767-1749)

The first person to use π to denote the ratio of the circumference to the perimeter of a circle

He worked in a counting house in London and was sent to the West Indies. Jones was interested in navigation and became a mathematics master on a man-of-war. On leaving the navy he became a mathematics teacher and in 1706 he published a text book based on his notes *Synopsis palmariorum matheseos*, or, *A New Introduction to the Mathematics*. (L. nn. 59, OO. l. 40). This was the first book to use the symbol π to denote the ratio of the circumference to the diameter of a circle. This occurs in two places in the book as can be seen on the copies of the pages on display. Jones was elected FRS in 1711 and was involved in the Newton-Leibniz controversy.

Figure 19: c.1796: William Jones



Palmariorum Mathefeos. 243

$$a = c + \frac{c^3}{6d^2} + \frac{3c^5}{40d^4}, \text{\&c. } A = C + \frac{C^3}{6d^2} + \frac{3C^5}{40d^4}, \text{\&c. (by 24)}$$

$$\text{Th. } C + \frac{C^3}{6d^2} +, \text{\&c.} = n \times c + \frac{c^3}{6d^2} +, \text{\&c.} = A$$

$$\text{Th. } C = \frac{nc}{1} + \frac{1-n^2}{2 \times 3d^2} c^2 \alpha + \frac{9-n^2}{4 \times 5d^2} c^2 \beta + \frac{25-n^2}{6 \times 7d^2} c^2 \gamma, \text{\&c.}$$

$$\text{38. Bec. t, } a = \left(\frac{rs}{s}\right) \frac{rs}{\sqrt{r^2 - s^2}} = (\text{if } a \text{ be } 30°) \frac{\frac{1}{2}}{\sqrt{\frac{3}{4}}} = \frac{1}{\sqrt{3}}$$

And $6a$, or $6 \times \overline{t - \frac{1}{3}t^3 + \frac{1}{5}t^5}$, &c. $= \frac{1}{2}$ Periphery (π)

But $6 \times \frac{1}{\sqrt{3}} = \frac{\sqrt{36}}{\sqrt{3}} = \sqrt{12} = 2\sqrt{3}$, and $t^2 = \frac{1}{3}$; Let

$\alpha = 2\sqrt{3}$, $\beta = \frac{1}{3}\alpha$, $\gamma = \frac{1}{3}\beta$, $\delta = \frac{1}{3}\gamma$, &c.

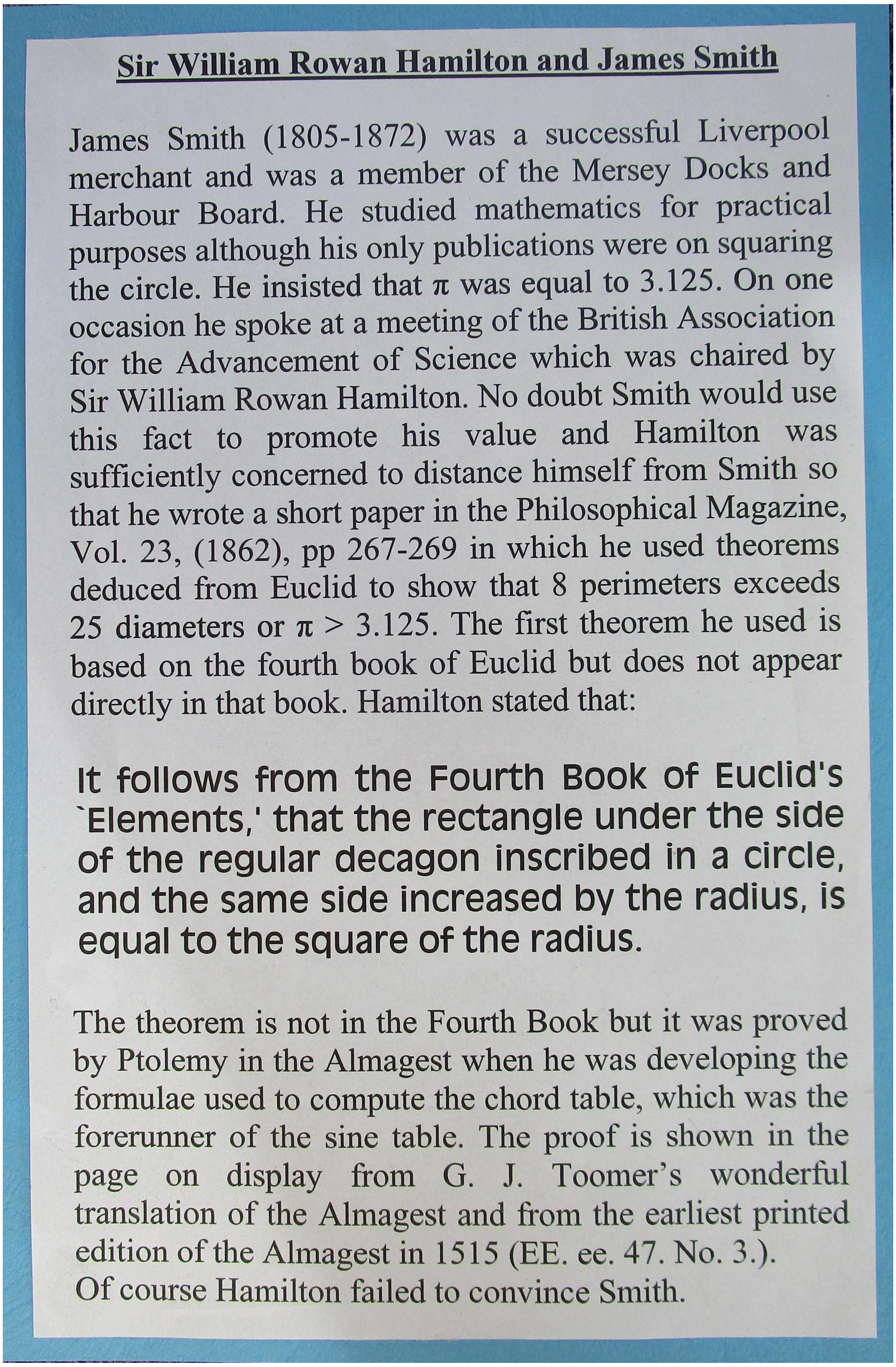
Then $\alpha - \frac{1}{3}\beta + \frac{1}{5}\gamma - \frac{1}{7}\delta + \frac{1}{9}\varepsilon$, &c. $= \frac{1}{2}\pi$, or

$$\alpha - \frac{1}{3}\frac{3\alpha}{9} + \frac{1}{5}\frac{\alpha}{9} - \frac{1}{7}\frac{3\alpha}{9^2} + \frac{1}{9}\frac{\alpha}{9^2} - \frac{1}{11}\frac{3\alpha}{9^3} + \frac{1}{13}\frac{\alpha}{9^3}, \text{\&c.}$$

Theref. *the (Radius is to* $\frac{1}{2}$ *Periphery, or) Diameter is to the Periphery, as* 1,000, &c. *to* 3.141592653.58979323 84.6264338327.9502884197.1693993751.0582097494. 4592307816.4062862089.9862803482.5342117067.9 +, True to above a 100 Places; as Computed by the Accurate and Ready Pen of the Truly Ingenious Mr. *John Machin*: Purely as an Inftance of the Vaft advantage *Arithmetical Calculations* receive from the *Modern Analyfis*, in a Subject that has bin of fo Engaging a Nature, as to have employ'd the Minds of the moft Eminent Mathematicians, in all Ages, to the Confideration of it. For as the exact Proportion between the *Diameter* and the *Circumference* can never be exprefs'd in Numbers; fo the Improvements of thofe Enquirers the more plainly appear'd, by how much the more Eafie and Ready, they render'd the Way to find a Proportion the neareft poffible: But the Method of Series (as improv'd by Mr. *Newton*, and Mr. *Halley*) performs this with great Facility, when compared with the Intricate and Prolix Ways of *Archimedes, Vieta, Van Ceulen, Metius, Snellius, Lansbergius*, &c. Tho' fome of them were faid to have (in this Cafe) fet Bounds to Human Improvements, and to have left no-

Ii 2

Figure 20: c.1796: William Jones Palmariorum Mathefos



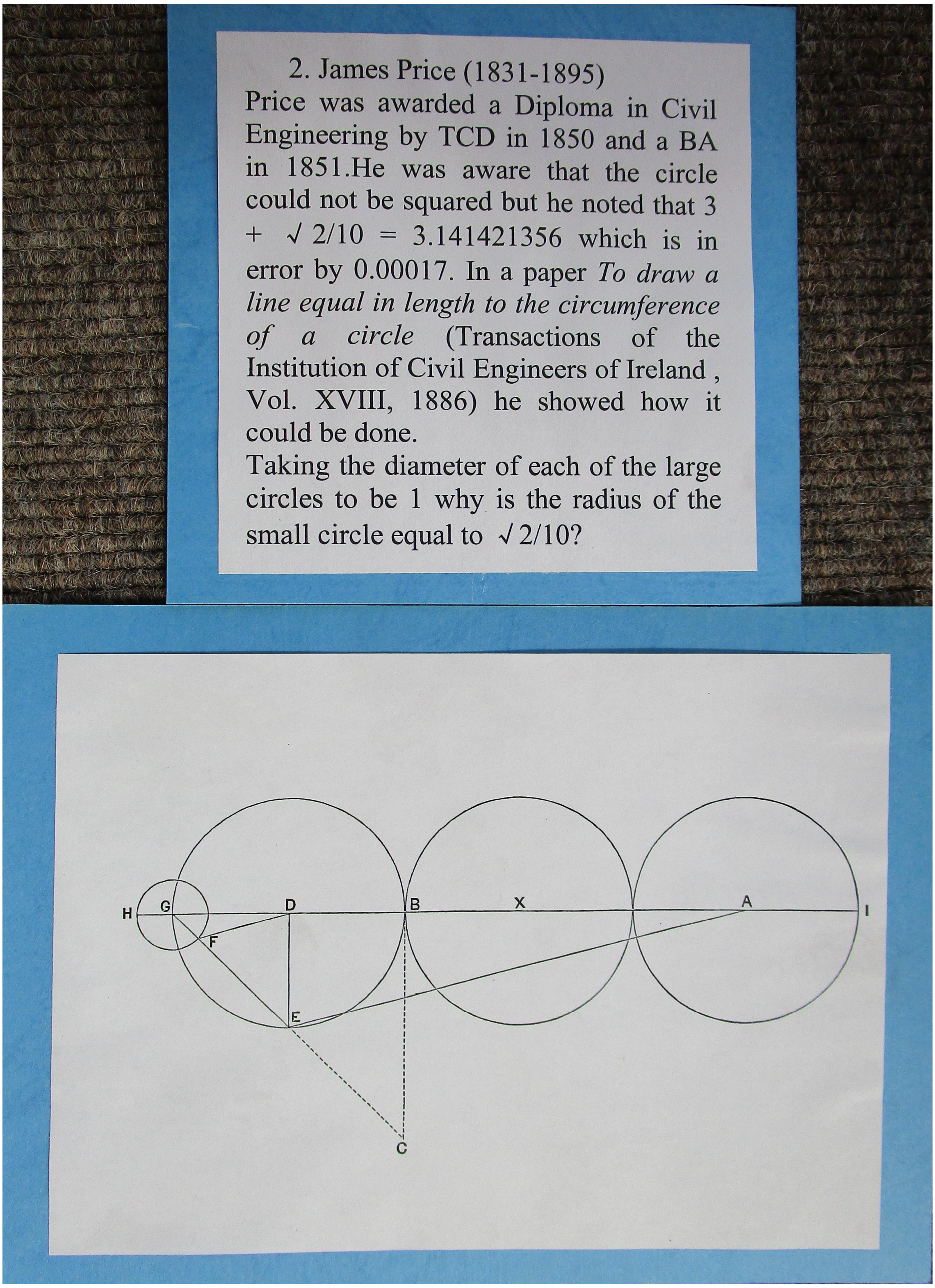
Sir William Rowan Hamilton and James Smith

James Smith (1805-1872) was a successful Liverpool merchant and was a member of the Mersey Docks and Harbour Board. He studied mathematics for practical purposes although his only publications were on squaring the circle. He insisted that π was equal to 3.125. On one occasion he spoke at a meeting of the British Association for the Advancement of Science which was chaired by Sir William Rowan Hamilton. No doubt Smith would use this fact to promote his value and Hamilton was sufficiently concerned to distance himself from Smith so that he wrote a short paper in the Philosophical Magazine, Vol. 23, (1862), pp 267-269 in which he used theorems deduced from Euclid to show that 8 perimeters exceeds 25 diameters or $\pi > 3.125$. The first theorem he used is based on the fourth book of Euclid but does not appear directly in that book. Hamilton stated that:

It follows from the Fourth Book of Euclid's `Elements,' that the rectangle under the side of the regular decagon inscribed in a circle, and the same side increased by the radius, is equal to the square of the radius.

The theorem is not in the Fourth Book but it was proved by Ptolemy in the Almagest when he was developing the formulae used to compute the chord table, which was the forerunner of the sine table. The proof is shown in the page on display from G. J. Toomer's wonderful translation of the Almagest and from the earliest printed edition of the Almagest in 1515 (EE. ee. 47. No. 3.).
Of course Hamilton failed to convince Smith.

Figure 21: c.1862: Hamilton

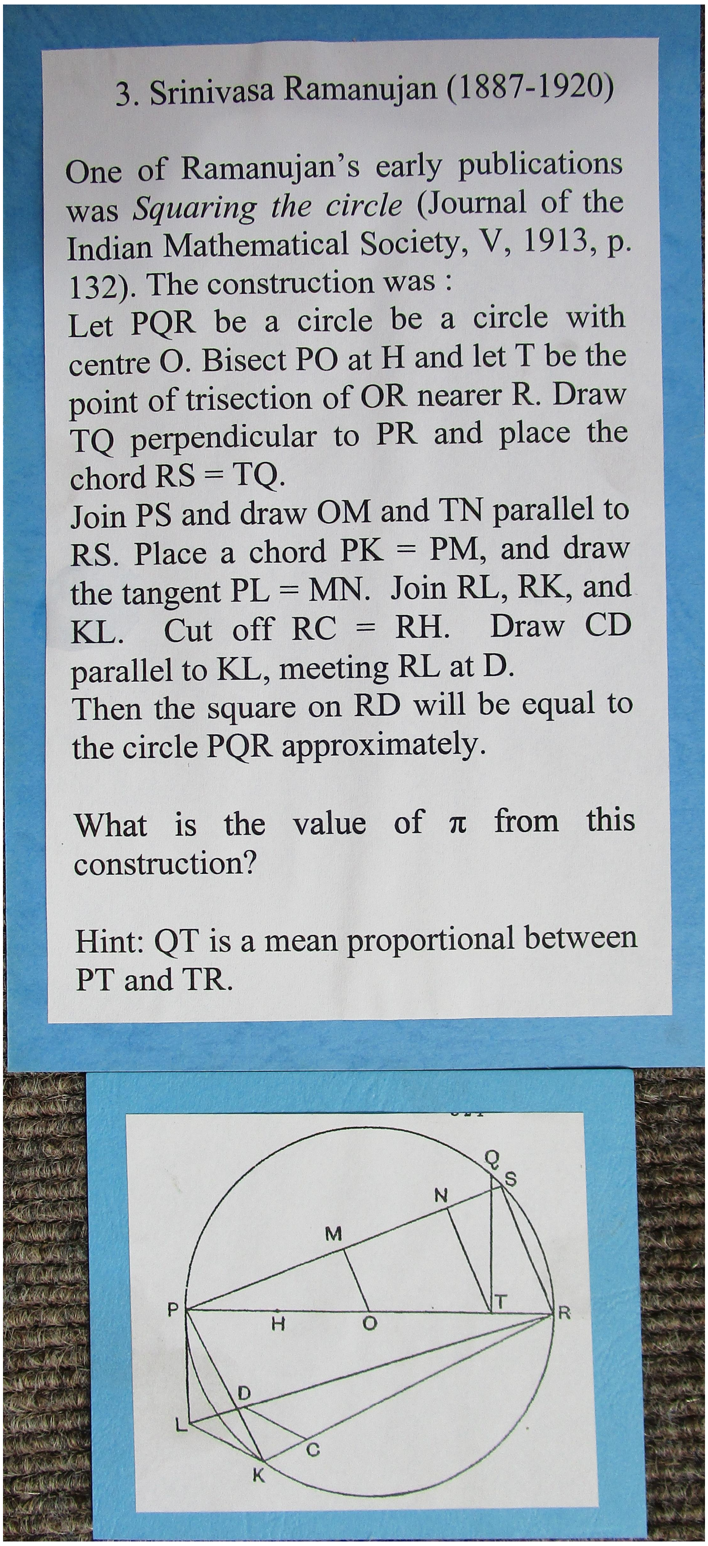


2. James Price (1831-1895)

Price was awarded a Diploma in Civil Engineering by TCD in 1850 and a BA in 1851.He was aware that the circle could not be squared but he noted that $3 + \sqrt{2}/10 = 3.141421356$ which is in error by 0.00017. In a paper *To draw a line equal in length to the circumference of a circle* (Transactions of the Institution of Civil Engineers of Ireland , Vol. XVIII, 1886) he showed how it could be done.

Taking the diameter of each of the large circles to be 1 why is the radius of the small circle equal to $\sqrt{2}/10$?

Figure 22: c.1886: James Price



3. Srinivasa Ramanujan (1887-1920)

One of Ramanujan's early publications was *Squaring the circle* (Journal of the Indian Mathematical Society, V, 1913, p. 132). The construction was :

Let PQR be a circle be a circle with centre O. Bisect PO at H and let T be the point of trisection of OR nearer R. Draw TQ perpendicular to PR and place the chord RS = TQ.

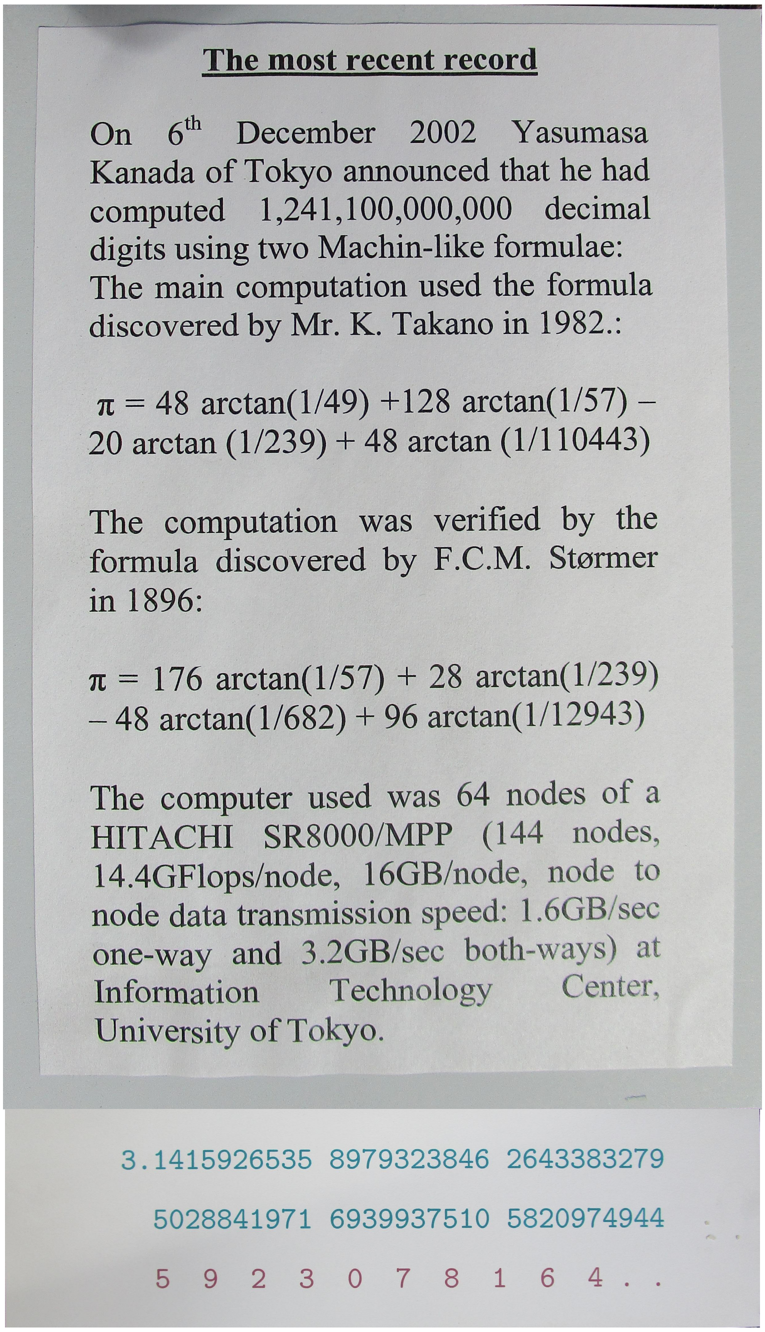
Join PS and draw OM and TN parallel to RS. Place a chord PK = PM, and draw the tangent PL = MN. Join RL, RK, and KL. Cut off RC = RH. Draw CD parallel to KL, meeting RL at D.

Then the square on RD will be equal to the circle PQR approximately.

What is the value of π from this construction?

Hint: QT is a mean proportional between PT and TR.

Figure 23: c.1913: Ramanujan



The most recent record

On 6th December 2002 Yasumasa Kanada of Tokyo announced that he had computed 1,241,100,000,000 decimal digits using two Machin-like formulae: The main computation used the formula discovered by Mr. K. Takano in 1982.:

$$\pi = 48 \arctan(1/49) + 128 \arctan(1/57) - 20 \arctan(1/239) + 48 \arctan(1/110443)$$

The computation was verified by the formula discovered by F.C.M. Størmer in 1896:

$$\pi = 176 \arctan(1/57) + 28 \arctan(1/239) - 48 \arctan(1/682) + 96 \arctan(1/12943)$$

The computer used was 64 nodes of a HITACHI SR8000/MPP (144 nodes, 14.4GFlops/node, 16GB/node, node to node data transmission speed: 1.6GB/sec one-way and 3.2GB/sec both-ways) at Information Technology Center, University of Tokyo.

3.1415926535 8979323846 2643383279

5028841971 6939937510 5820974944 . .

5 9 2 3 0 7 8 1 6 4 . .

Figure 24: c.2002: Yasumasa, long value of π

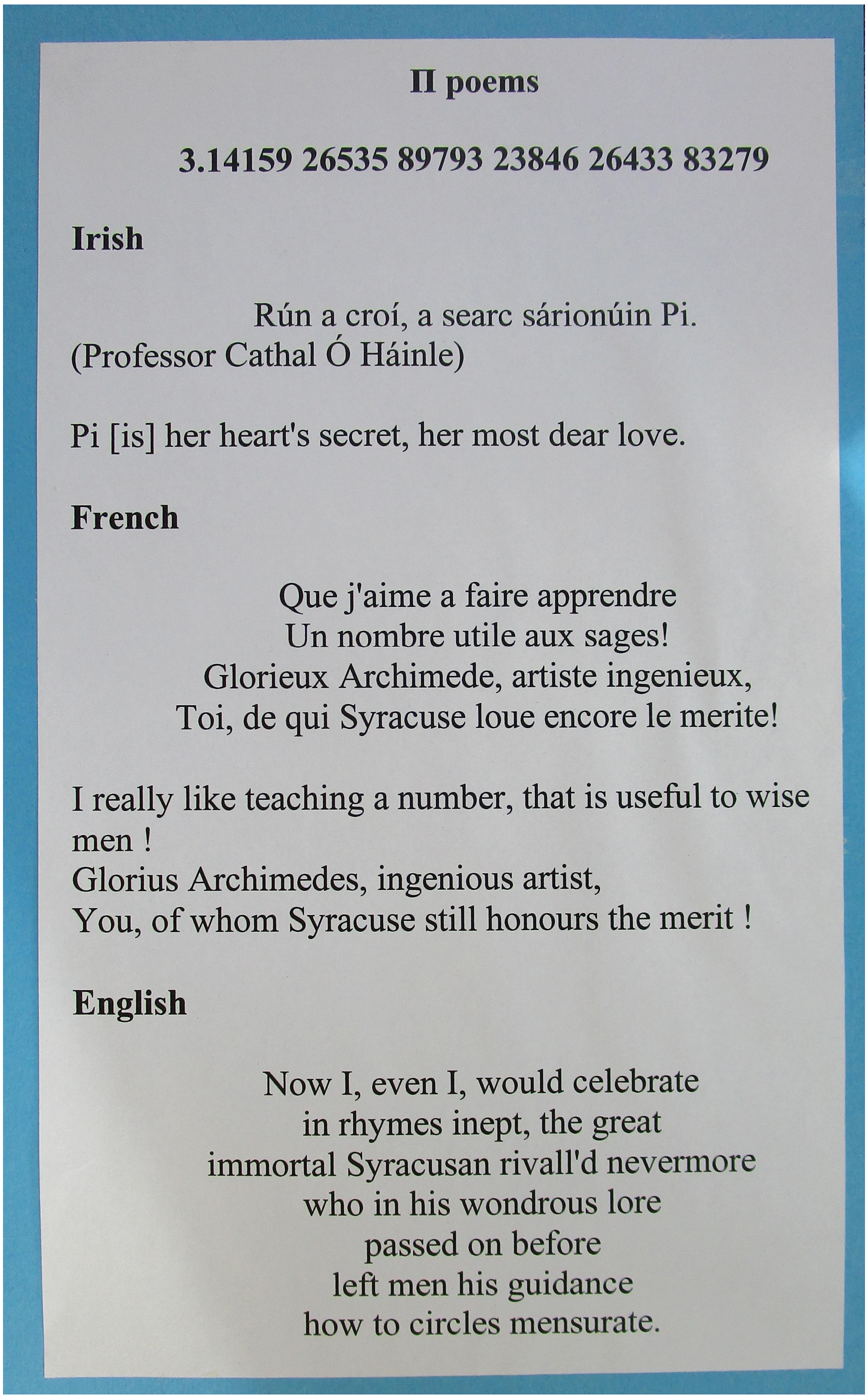


Figure 25: π poems

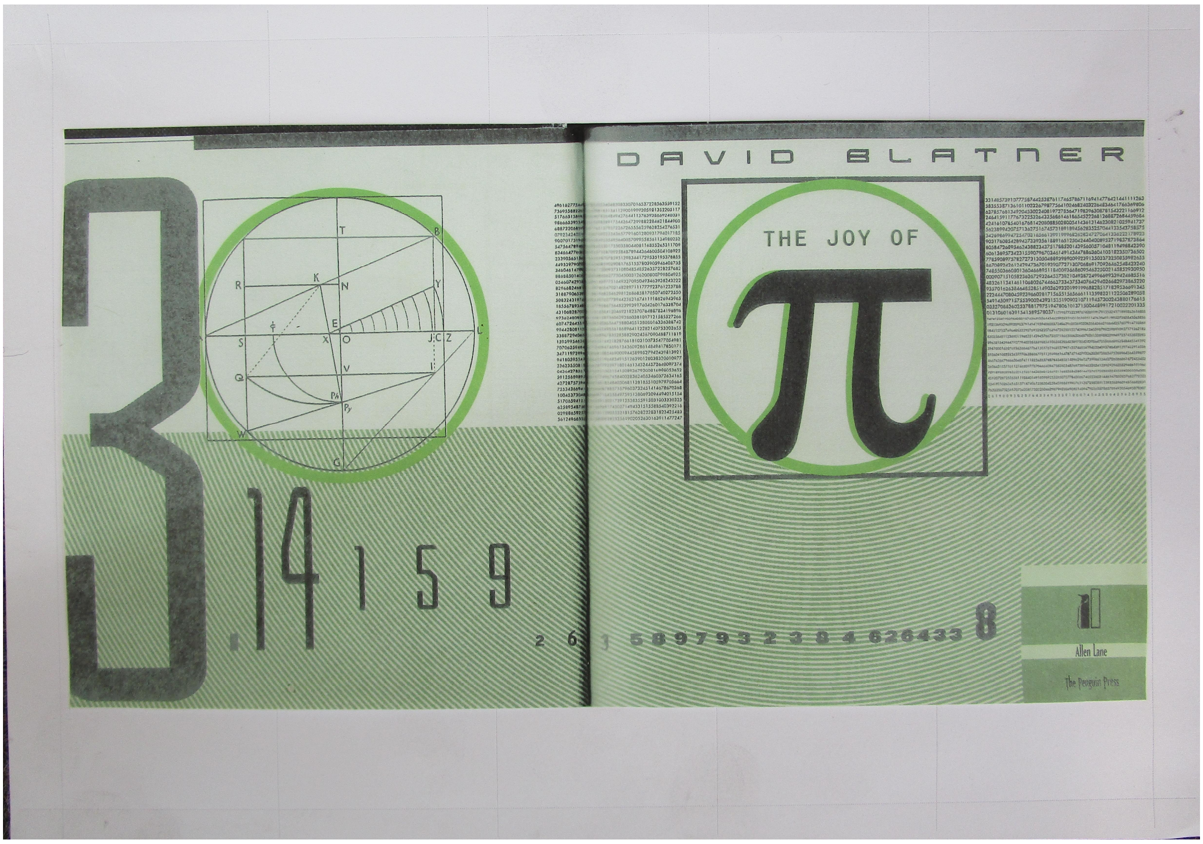


Figure 26: David Blatner, "The Joy of π"

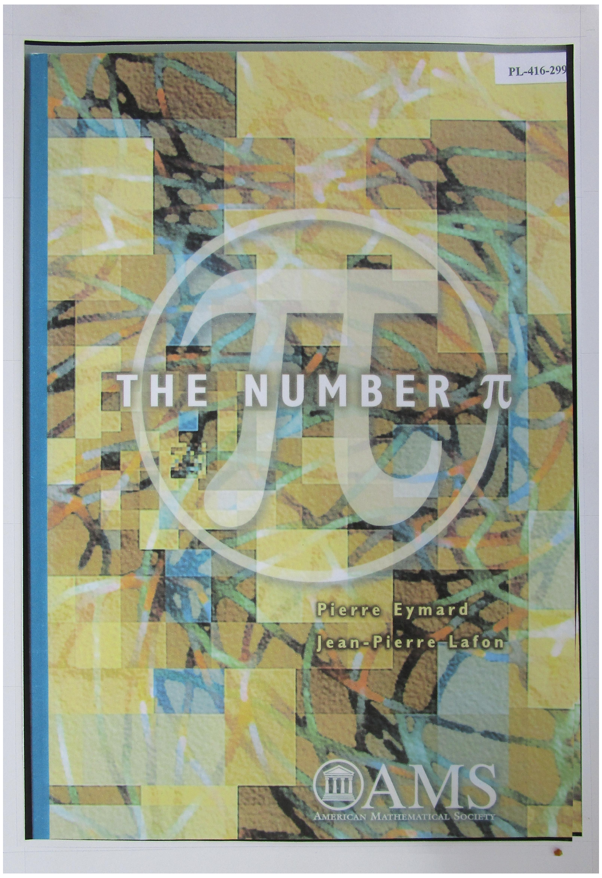


Figure 27: Pierre Eymard, Jean-Pierre Lafon, "The Number π"

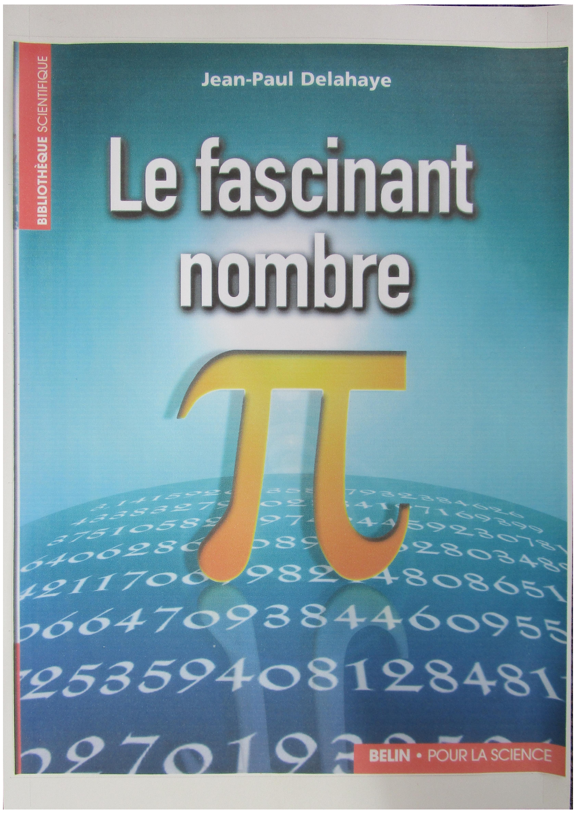


Figure 28: Jean-Paul Delahaye, "Le fascinant nombre π"

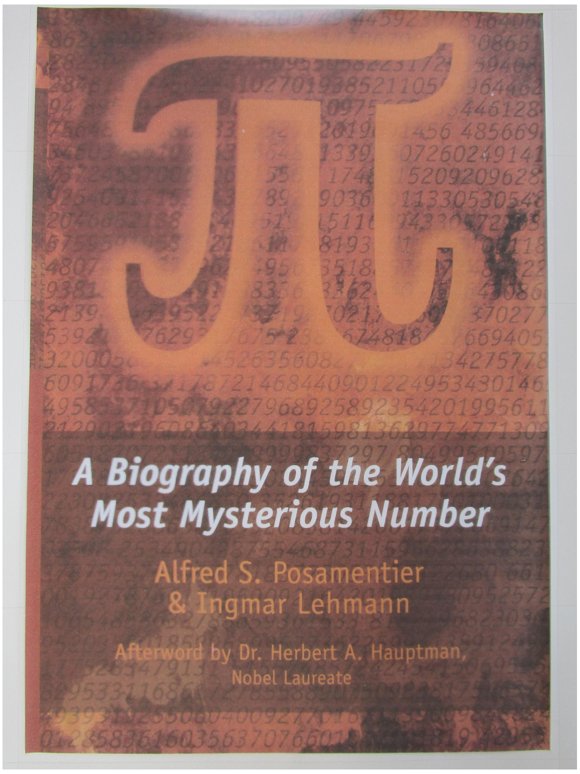


Figure 29: Alfred S. Posamentier & Ingmar Lehmann, "π, A Biography of the World's Most Mysterious Number"

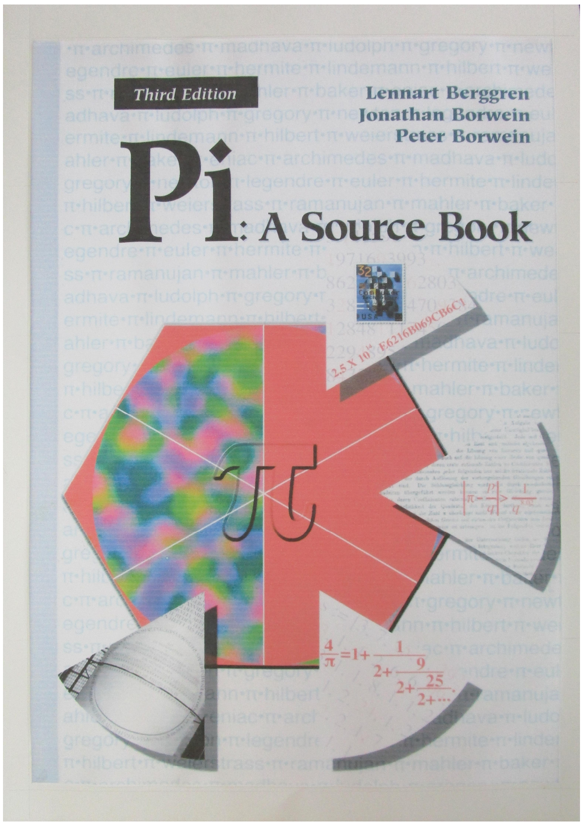


Figure 30: Lennart Berggren, Jonathan Borwein, Peter Borwein, "π: A Source Book"

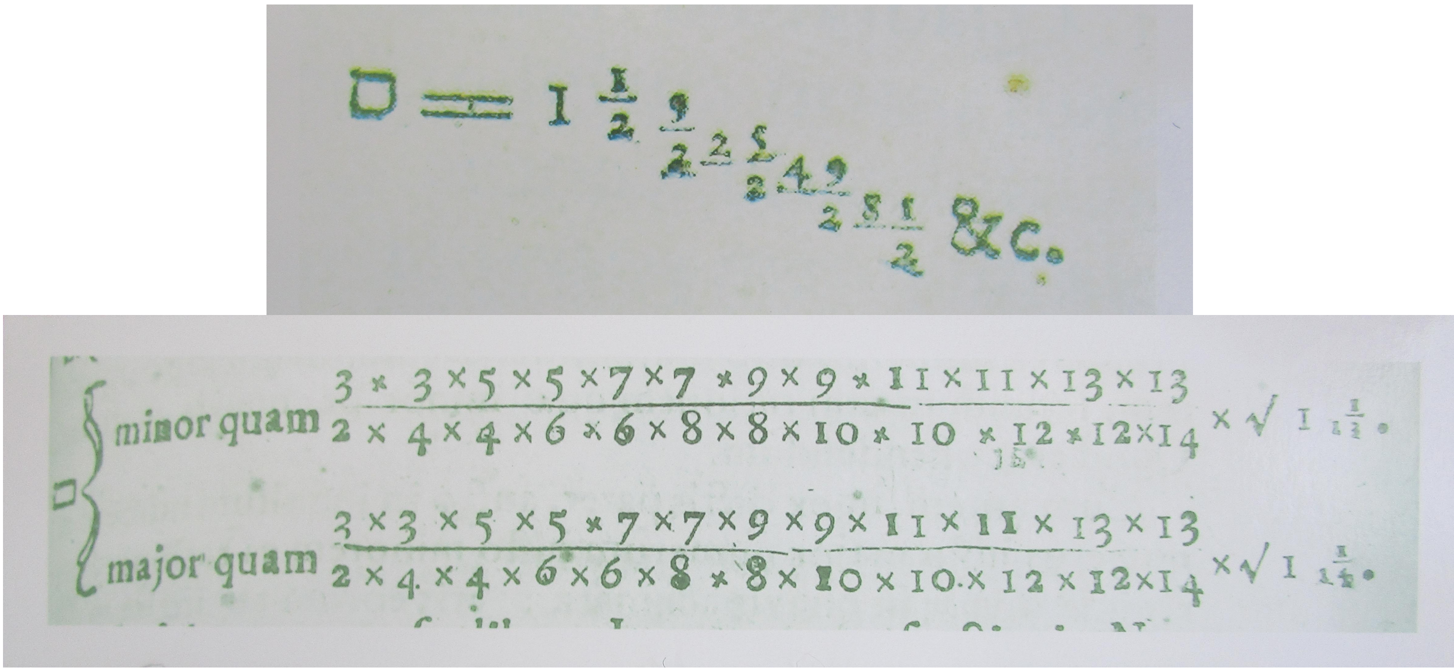


Figure 31: Two expressions of π